Response to the First ACT Competition on Global Trajectory Optimisation

held by the

Advanced Concepts Team

of the

European Space Agency

as a precursor to its

2006 Global Trajectory Optimisation Workshop

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Team 11

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Introduction

Here we present a summary of our results for a theoretical trajectory optimisation problem, posed by the Advanced Concepts Team of the European Space Agency in the framework of the 2006 Global Trajectory Optimisation Workshop which they will host. The posed problem constitutes the First Global Trajectory Optimisation Competition, which is a precursor to the Workshop. The details regarding the Workshop and Competition may be found on the world wide web at Ref. 5.

Refs. 5 and 6 provide the details of the problem statement. The problem is essentially one of celestial mechanics and mathematics, rather than of engineering. In brief, the goal is to design a theoretical optimal low-thrust trajectory departing from the Earth and impacting an asteroid. The cost function is the product of the final spacecraft mass and the absolute value of the dot product of the arrival v-infinity vector with the asteroid's velocity at impact.

Our search for the globally optimal trajectory was conducted in two phases. First, a rough global search was made over a very large part of the solution space. Second, the most promising regions found in the rough search were examined more carefully with a local optimisation method. These two phases are described in detail below. The trajectory with the best objective function, based on our search, is presented thereafter. It should be noted that the bounds on the rough global search were placed based on mission design experience, as described below. Also, the search is not fully automated: for example, there is no automatic handoff from the global search algorithm to the local optimisation algorithm.

Rough global search

Bounding of the Search Space

Let us first examine how we will limit the rough global search to a sensible, yet still very large, portion of the solution space. It is clear that the cost function will be increased by 1) increasing the final spacecraft mass, 2) increasing the arrival v-infinity, 3) arriving with a v-infinity that is more collinear with the velocity of the asteroid, or 4) arriving when the asteroid's speed is larger. The first factor indicates that the trajectory should employ gravity assists *in lieu* of thrust wherever possible. The second, third and fourth factors indicate that the spacecraft should, if possible, be on a retrograde orbit and impact the asteroid when both the spacecraft and the asteroid are at or near periapsis. Thus, the trajectory should employ some sequence of gravity assists at the two gas giants at our disposal.

The first question is how to reach the gas giants. Given the very low departure v-infinity of 2.5 km/s, to avoid excessive thrusting, it is likely best to use Venus and Earth as the first and second gravity assist bodies, respectively. Other combinations for the first two gravity assists are not as energetically attractive. Furthermore, since the goal is to enlarge the orbit, a return to Venus is not as advantageous as a return to the Earth.

The second question is what sequence of flybys to use at Jupiter and Saturn. Given their long periods and the flight time limit of 30 years, only a handful of combinations are practical.

Taking the above two questions into account, we examined the following "paths" (i.e. sequence of gravity assist bodies) for the whole launch window. The planets are represented by their initial letter, with M being for Mars:

VEEJS
VEEJSJ
VEESJ
VEMEJSJ
VEMEMJSJ
VEEES
VEEEJS
VEEESJ
VEEEJSJ
VEEJESJ
VEEJVES
VEEJVESJ
VEMJJ
VEMJS
VEMJSJ

The paths with inner-solar-system flybys after flybys at Jupiter were examined for the outside chance that the increased v-infinity afforded upon return to the gas giants would be helpful.

This list of paths is by no means exhaustive; there may well be other paths that offer better performance. As a side note, if there were no flight time limit, or only a very large one, the gravity-assist combinations and the trajectory as a whole would take on a very different character.

Searching in the Search Space

With the software STOUR-LTGA, described fully in Refs. 1 and 2, automated grid searches for trajectories were made for each of the above paths, using low thrust only on the Earth to Venus leg. In STOUR-LTGA, low-thrust arcs are computed using a shape-based approach, where the shape is taken as the exponential sinusoid. To compensate for the likely non-optimality of following the given shape, a slightly higher thrust acceleration than that actually available is permitted, and a slightly higher launch v-infinity is permitted as well. Other constraints are also relaxed a little, to compensate for the roughness of the search. The grid search is over launch date and over launch v-infinity values. The ballistic legs of the trajectory are computed exhaustively, that is, all possible ballistic trajectories satisfying the relaxed constraints are computed. For the low-thrust leg, a judicious selection, based on various criteria, is made to obtain a finite number of trajectories from the continuum of solutions available. Each of the selected solutions for the low-thrust leg is passed on to the exhaustive ballistic computation engine. For some paths, when a ten-day step size in the launch date and two launch v-infinities were used, up to 30000 trajectories were found.

Of the hundreds of thousands of trajectories found in total using STOUR-LTGA, a handful were selected on the basis of very high arrival v-infinity (on the order of 50 km/s), or of very high cost function. As a visual example, for the VEEJS path, the cost function is plotted versus the launch date in Fig. 1.

These promising solutions were then handed over to the local optimisation program, named MALTO.

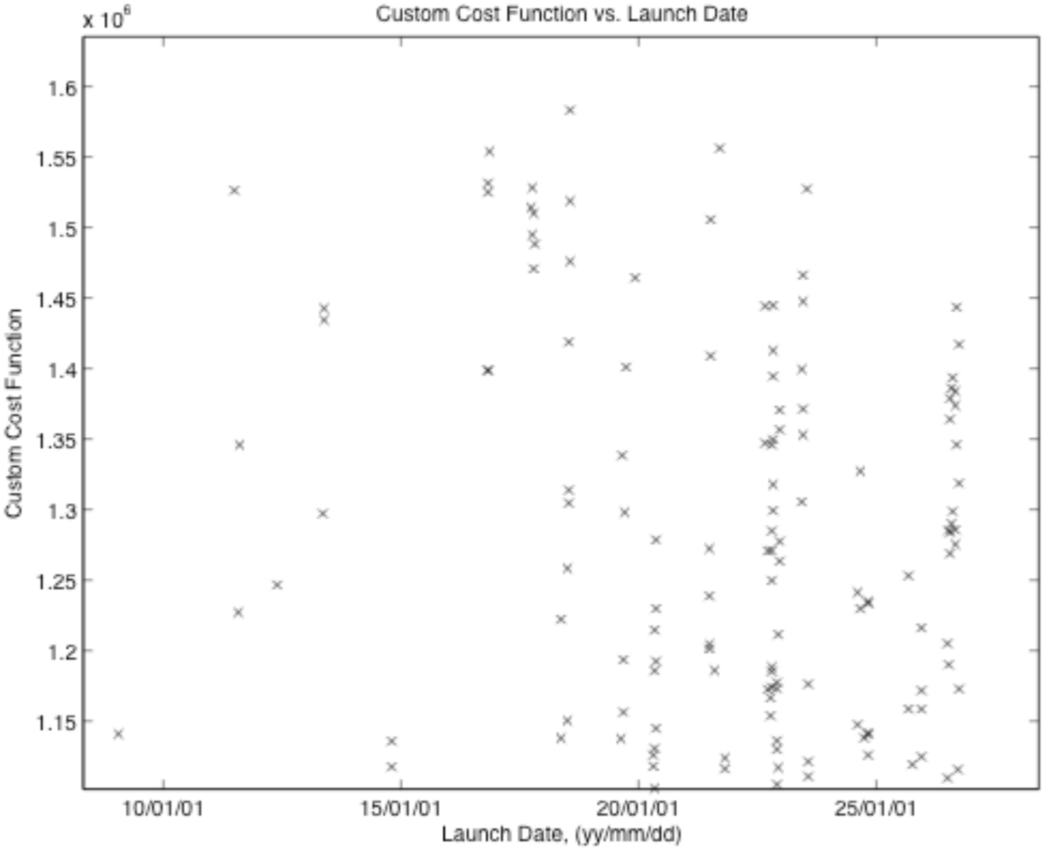

Fig. 1 Cost function versus launch date (zoomed to highest values of cost function), for the VEEJS path

Local optimisation

The local optimisation program, MALTO, is based on the algorithm formulated by Sims and Flanagan in Ref. 3. In this formulation, the low-thrust arcs are modelled as a series of impulsive velocity increments (Delta-Vs), and the flybys are modelled as instantaneous turns of the v-infinity vector. Each leg (i.e. from one flyby body or control point to the next one in time) is split into a number of segments, as shown in Fig. 2. Targeting is done by means of a match point, which occurs after a specified number of segments from the first control point. The trajectory is propagated forward in time from the first control point to the match point, and backwards in time from the next control point to the match point. Trajectory propagation is conic, except that at the temporal midpoint of the segments a discontinuity in the velocity (Delta-V) is allowed. The maximum magnitude of the Delta-V is equal to the available thrust acceleration multiplied by the duration of the segment. From the rocket equation, the mass drop corresponding to the Delta-V is computed. When a sufficient number of segments is used (for near-circular orbits, 20 to 30 segments per revolution is normally sufficient), this formulation provides an excellent approximation to an actual low-thrust arc.

The optimisation variables are the magnitude and direction of the impulses; the launch, flyby and arrival times; the incoming and outgoing v-infinity vectors at the flyby bodies; the outgoing/incoming v-infinity vectors at the launch/arrival body; and also the initial spacecraft mass (for the outside chance that the benefit of increased thrust acceleration could outweigh the penalty of reduced mass; initial mass would be reduced by dumping some propellant instantaneously). The optimisation engine used is SNOPT (Ref. 4), which is based on the sequential quadratic programming method. SNOPT finds locally optimal solutions which satisfy the non-linear constraints. Appropriate scaling is used for the variables and analytic derivatives are used.

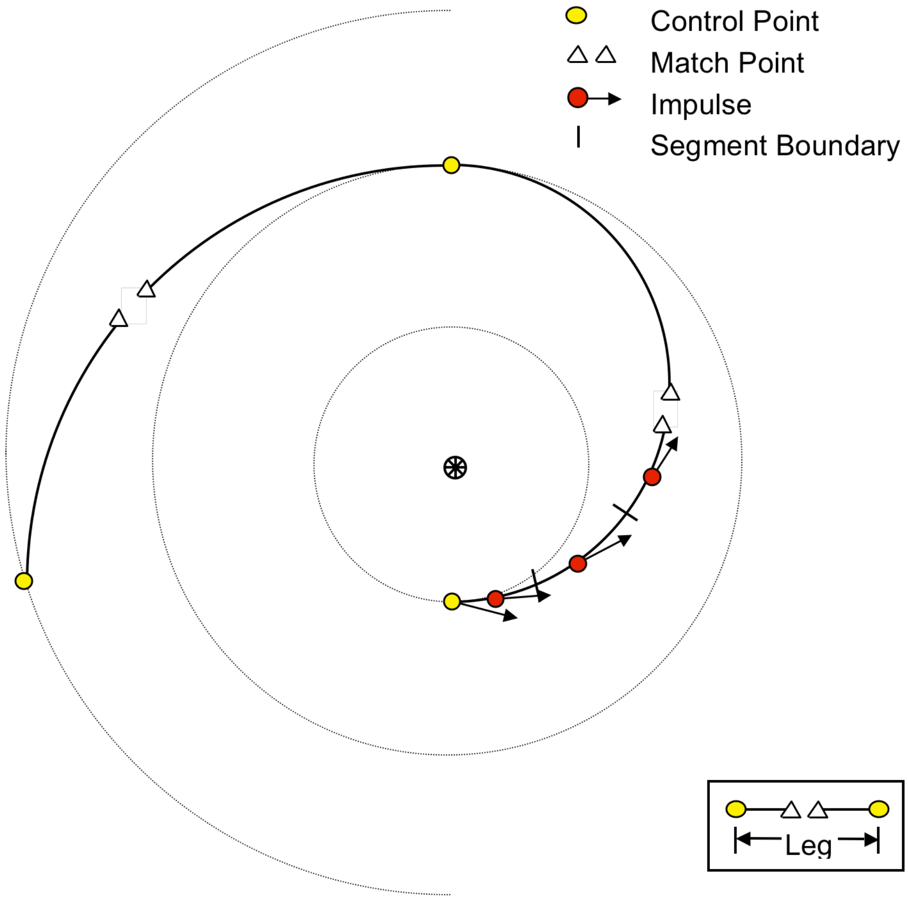

Fig. 2 Trajectory Modelling in Local Optimiser, MALTO (after Sims and Flanagan, Ref. 3)

Optimal Trajectory

Of the several trajectories optimised based on the best candidates from the rough global search, the following trajectory had the highest cost function.

Body	**Date** (yyyymmdd)	**JD**	**V-infinity** (km/s)	**Radius** (km)	**S/C Mass** (kg)
Earth Launch	20240820	2460542.8235	2.500	n/a	1500.00
Venus	20280219	2461820.8074	7.041	6351	1442.93
Earth	20300926	2462770.8083	11.860	10132	1442.93
Earth	20331228	2463959.8889	11.822	27207	1442.92
Earth	20381019	2465715.5769	11.868	6890	1442.92
Jupiter	20400217	2466201.8449	14.411	1216508	1442.92
Saturn	20410613	2466684.2978	15.298	90909	1442.92
Jupiter	20500601	2469958.5332	25.109	11228358	1442.92
2001 TW229	20511126	2470502.0494	52.662	n/a	1442.91

Total flight time (days)	9959.23
Total duration of thrust (days)	404.96
Objective function (kg(km/s)^2)	1851322.0

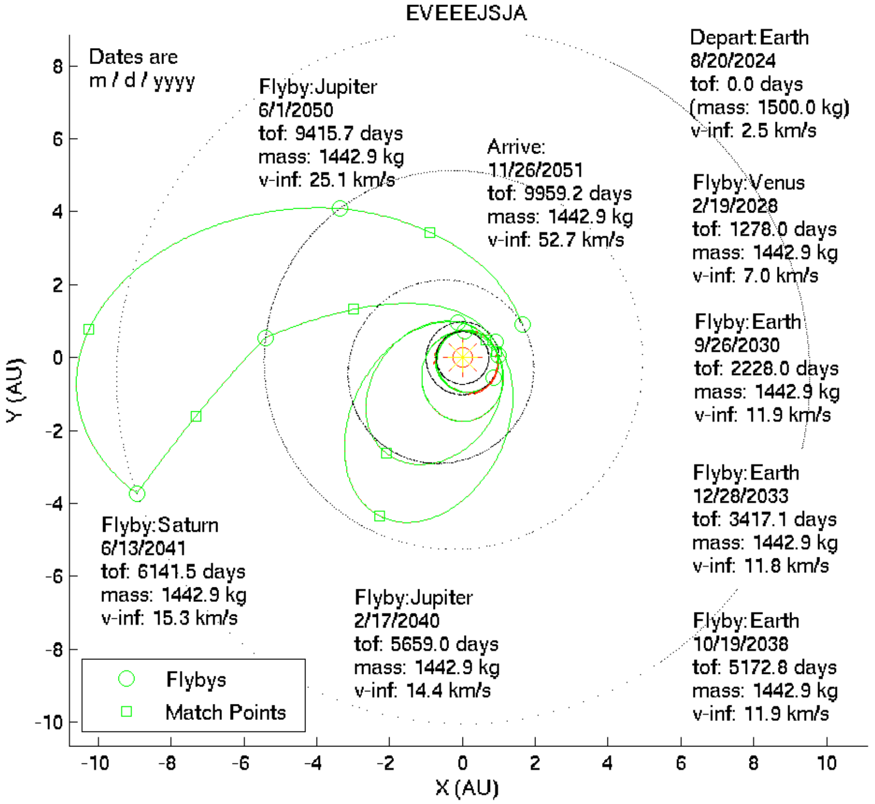

Fig. 2 Optimal trajectory found (Earth – VEEEJSJ – Asteroid 2001 TW229)

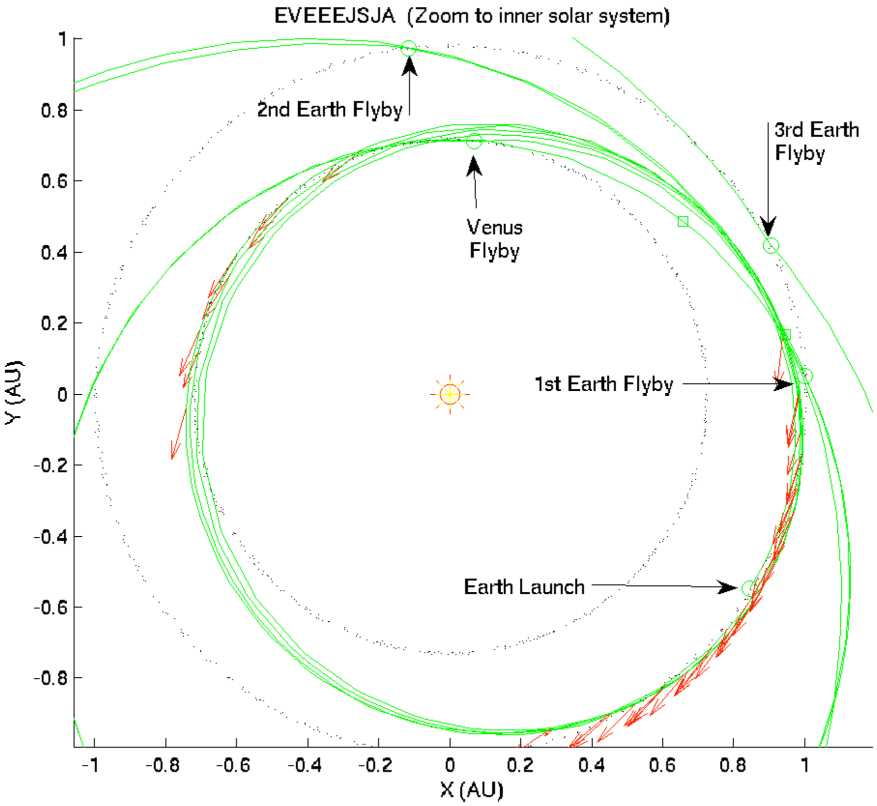

Fig. 3 Zoom view of optimal trajectory of Fig. 2

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