Orbital Maneuvers

Chapter 6

Maneuvers

- We assume our maneuvers are "instantaneous"
- ΔV changes can be applied by changing the magnitude "pump" or by changing the direction "crank"
- Rocket equation: relating ΔV and change in mass

$$\boxed{\Delta V = g_0 I_{SP} \ln\left(\frac{m_{initial}}{m_{final}}\right)}$$

or

$$m_{final} = \frac{m_{initial}}{\exp\left(\frac{\Delta V}{g_0 I_{SP}}\right)}$$

where $g_0 = 9.806\ m/s^2$

I_{sp}

- I_{sp} is the specific impulse (units of seconds) and measures the performance of the rocket

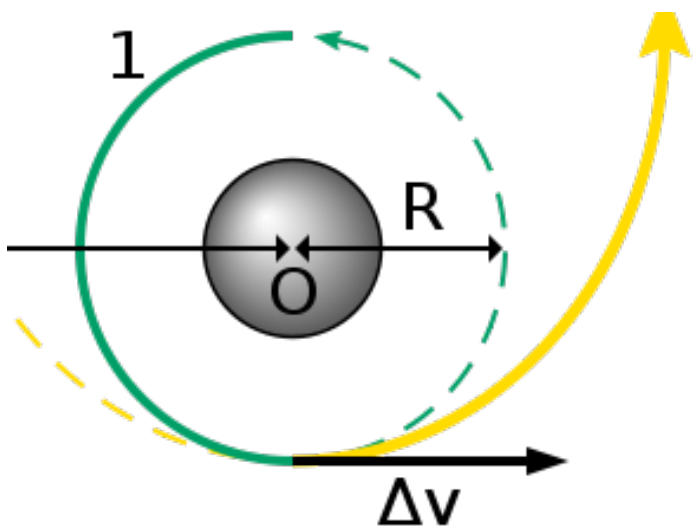
Table 6.1 Some Typical Specific Impulses

Propellant	I_{sp} (seconds)
Cold gas	50
Monopropellant hydrazine	230
Solid propellant	290
Nitric acid/monomethylhydrazine	310
Liquid oxygen/liquid hydrogen	455
Ion propulsion	>3000

ΔV

- ΔV ("delta"-V) represents the instantaneous change in velocity (from current velocity to the desired velocity).

$$\Delta V = \left|\mathbf{V}_2 - \mathbf{V}_1\right|$$



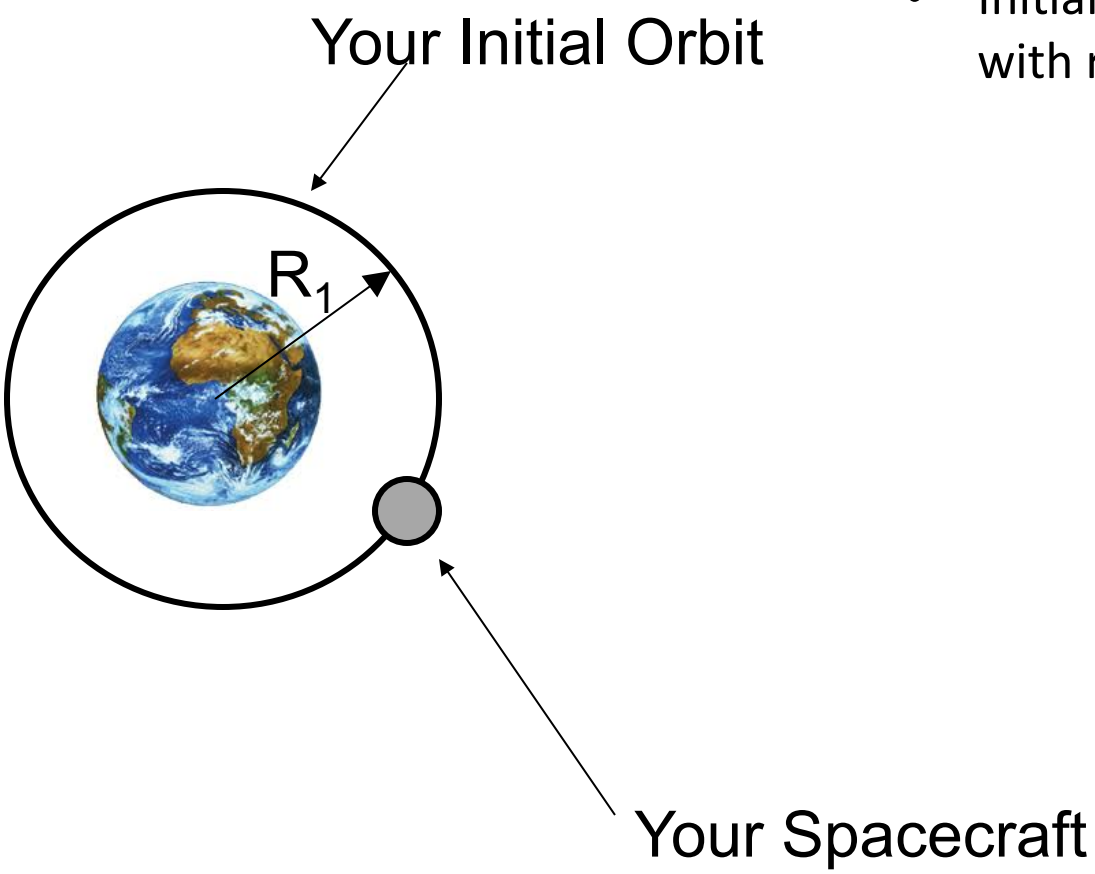
Circular Orbit Velocity

$$V_C = \sqrt{\frac{\mu}{R}}$$

Elliptical Orbit Velocity

$$V_E = \sqrt{\mu\left(\frac{2}{R} - \frac{1}{a}\right)}$$

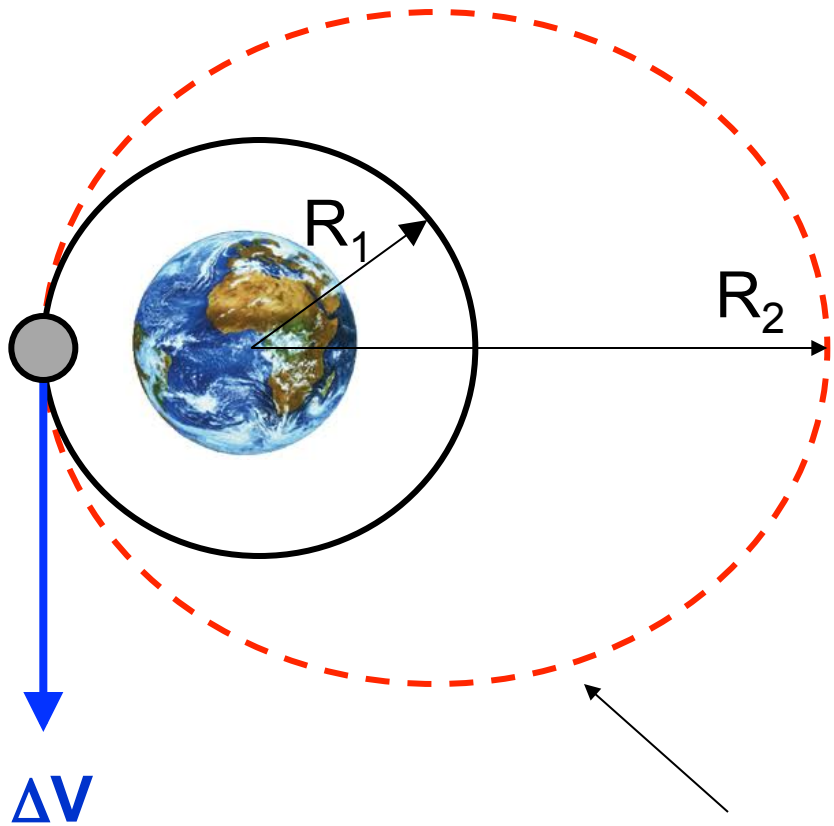
Tangent Burns



- Initially you are in a circular orbit with radius R_1 around Earth

$$V_{C_1} = \sqrt{\frac{\mu}{R_1}}$$

Tangent Burns



Your new orbit after the ΔV burn

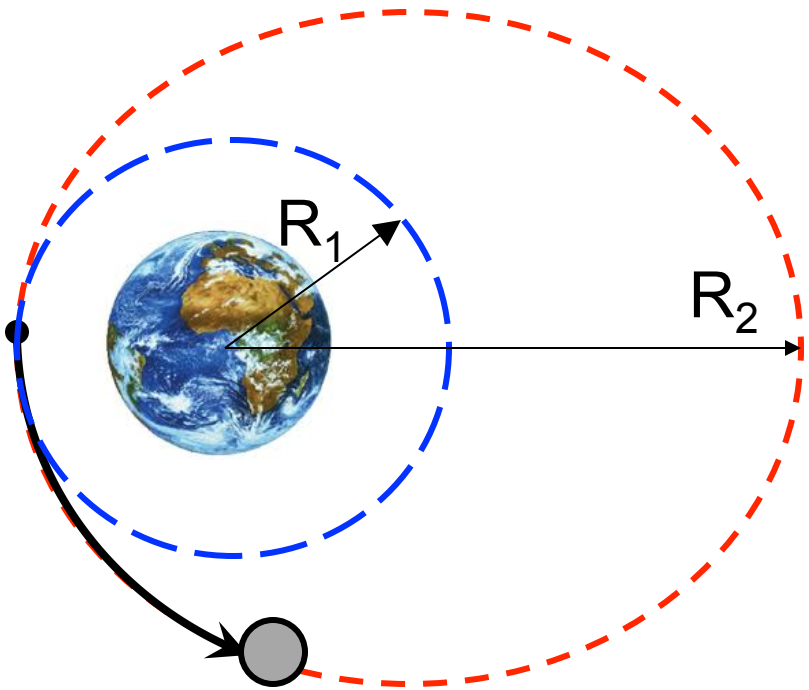
- Initially you are in a circular orbit with radius R_1 around Earth

$$V_{C_1} = \sqrt{\frac{\mu}{R_1}}$$

- You perform a burn now which puts in into the red-dotted orbit. So you perform a ΔV.

$$\Delta V = \left| V_{E_1} - V_{C_1} \right|$$

Tangent Burns



- Initially you are in a circular orbit with radius R_1 around Earth

$$V_{C_1} = \sqrt{\frac{\mu}{R_1}}$$

- You perform a burn now which puts in into the red-dotted orbit. So you perform a ΔV.

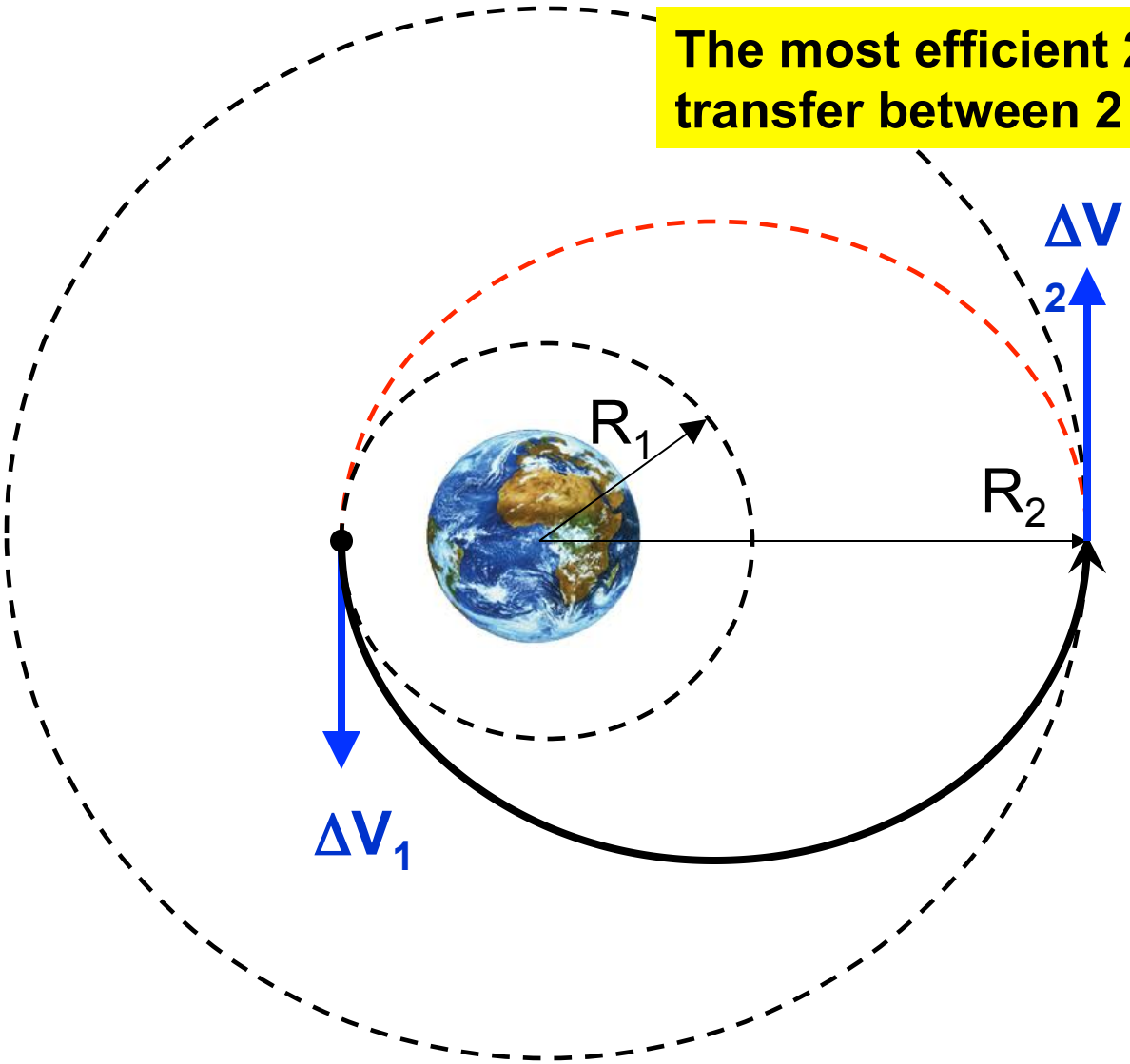
$$\Delta V = \left| V_{E_1} - V_{C_1} \right|$$

where

$$V_{E_1} = \sqrt{\mu\left(\frac{2}{R_1} - \frac{1}{a}\right)} \quad \text{and} \quad a = \frac{1}{2}\left(R_1 + R_2\right)$$

Hohmann Transfer

The most efficient 2-burnmaneuver to transfer between 2 co-planar circular orbit



$$\Delta V = \Delta V_1 + \Delta V_2$$

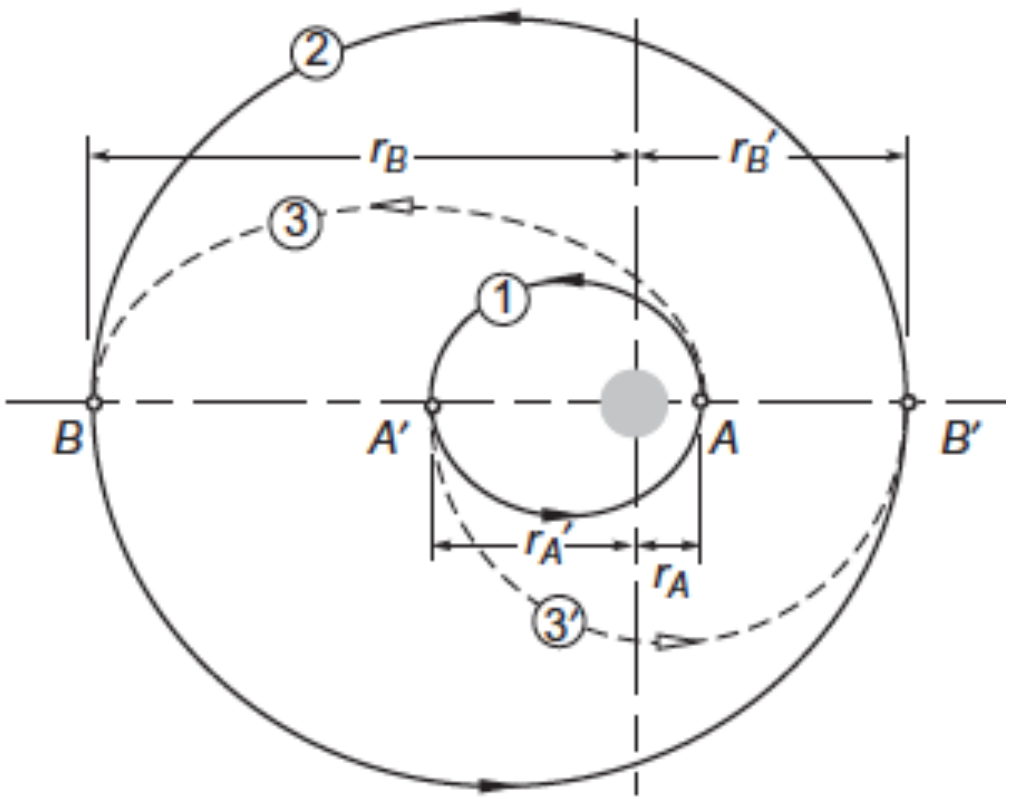
$$\Delta V_1 = \sqrt{\frac{\mu}{R_1}}\left(\sqrt{\frac{2R_2}{R_1+R_2}} - 1\right)$$

$$\Delta V_2 = \sqrt{\frac{\mu}{R_2}}\left(1 - \sqrt{\frac{2R_1}{R_1+R_2}}\right)$$

$$TOF = \frac{1}{2}P = \pi\sqrt{\frac{a^3}{\mu}}$$

Hohmann Transfer (non-circular)

- Another way to view it is using angular momentum



$$h = \sqrt{2\mu\left(\frac{r_a r_p}{r_a + r_p}\right)}$$

$$\Delta V_{total_3} = \Delta V_A + \Delta V_B$$

$$\Delta V_{total_3'} = \Delta V_{A'} + \Delta V_{B'}$$

Hohmann Transfer (non-circular)

$$h_1 = \sqrt{2\mu\left(\frac{r_A r_{A'}}{r_A + r_{A'}}\right)}$$

$$h_2 = \sqrt{2\mu\left(\frac{r_B r_{B'}}{r_B + r_{B'}}\right)}$$

$$h_3 = \sqrt{2\mu\left(\frac{r_A r_B}{r_A + r_B}\right)}$$

$$h_{3'} = \sqrt{2\mu\left(\frac{r_{A'} r_{B'}}{r_{A'} + r_{B'}}\right)}$$

$$V_A\big|_1 = h_1 / r_A \qquad V_A\big|_3 = h_3 / r_A$$

$$V_B\big|_1 = h_2 / r_B \qquad V_B\big|_3 = h_3 / r_B$$

$$V_{A'}\big|_1 = h_1 / r_{A'} \qquad V_{A'}\big|_{3'} = h_{3'} / r_{A'}$$

$$V_{B'}\big|_1 = h_2 / r_{B'} \qquad V_{B'}\big|_{3'} = h_{3'} / r_{B'}$$

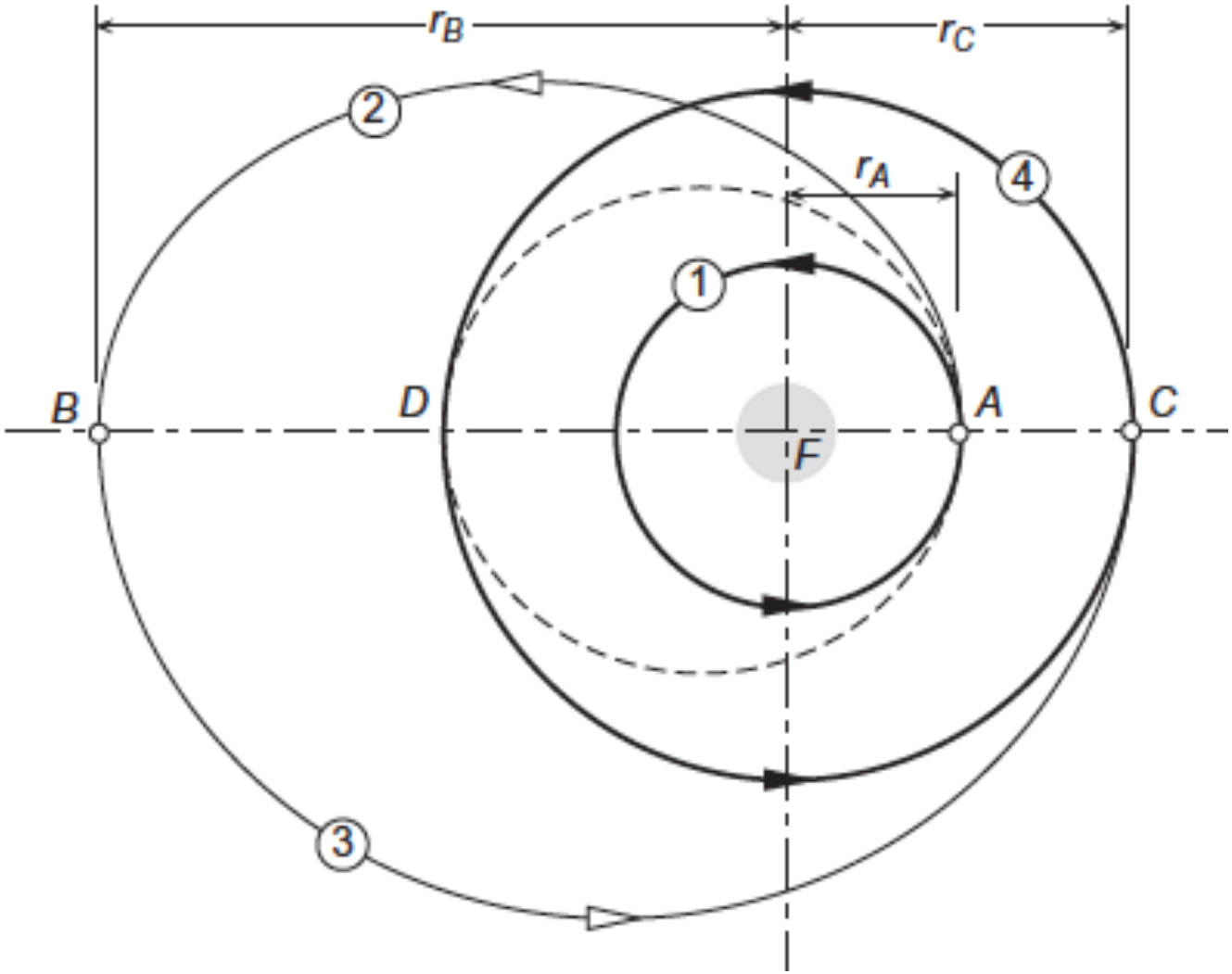
$$\Delta V_A = abs\left(V_A\big|_3 - V_A\big|_1\right)$$

$$\Delta V_{A'} = abs\left(V_{A'}\big|_3 - V_{A'}\big|_1\right)$$

$$\Delta V_B = abs\left(V_B\big|_2 - V_B\big|_3\right)$$

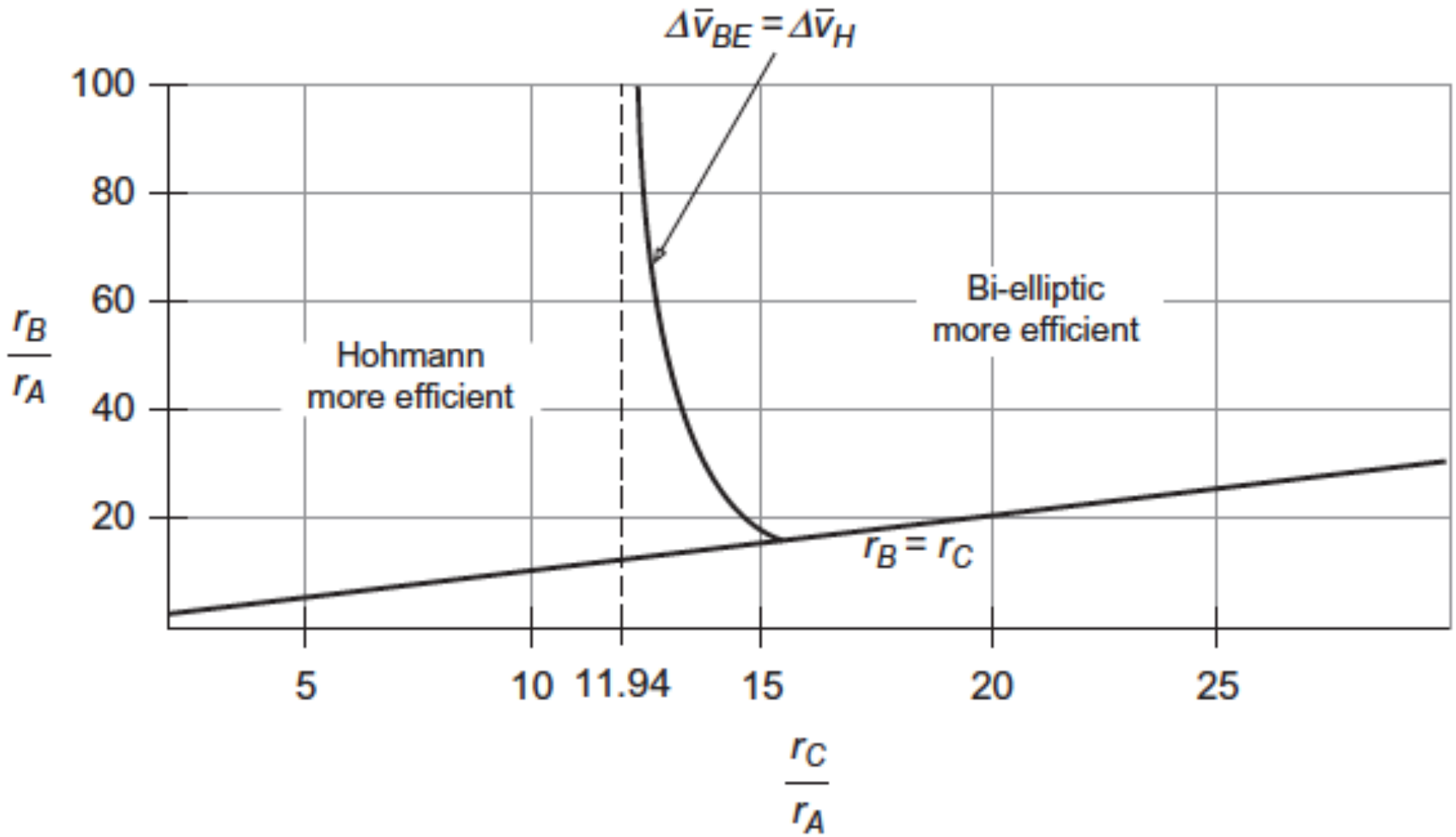
$$\Delta V_{B'} = abs\left(V_{B'}\big|_2 - V_{B'}\big|_{3'}\right)$$

Bi-Elliptic Transfer

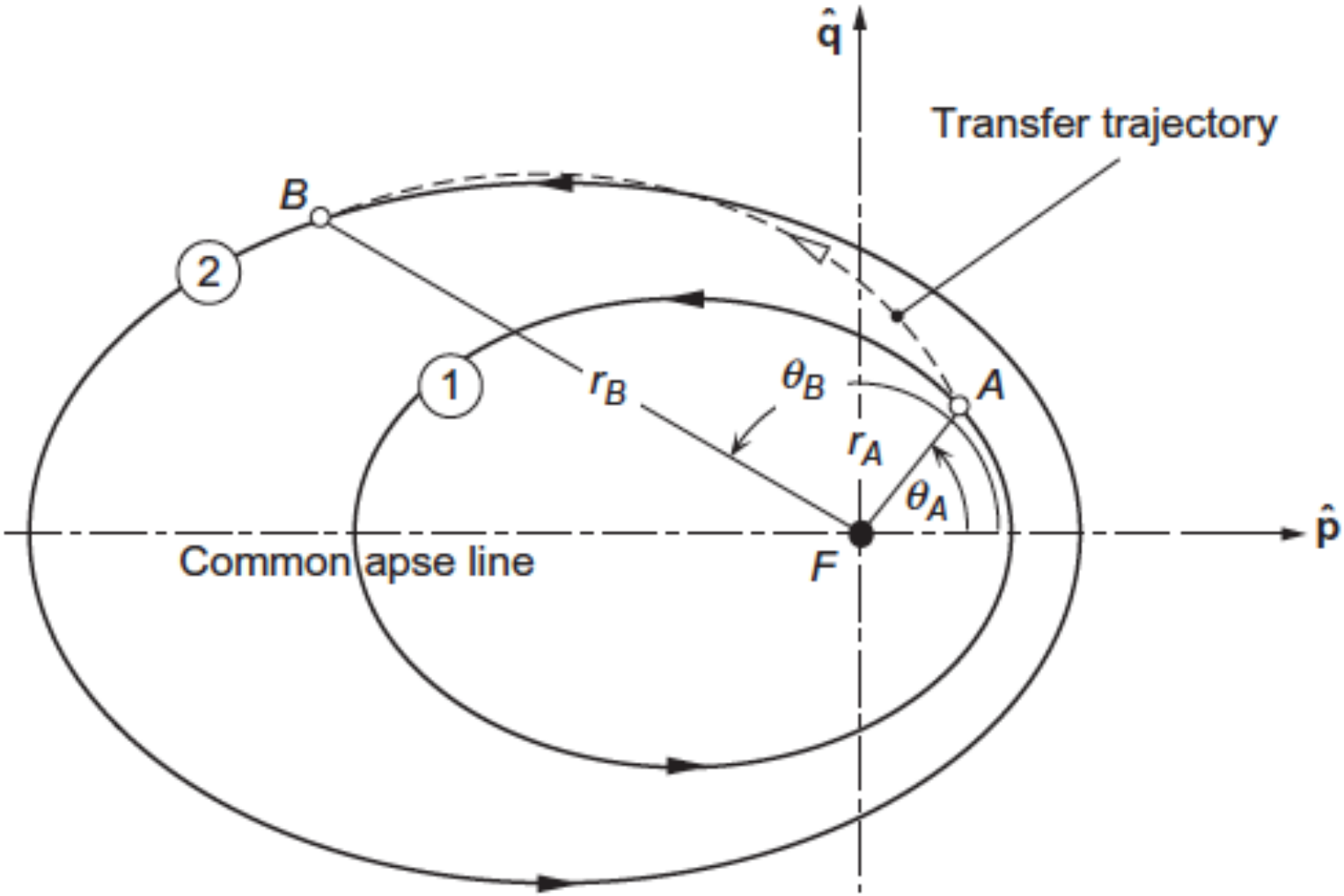


Bi-Elliptic Transfer

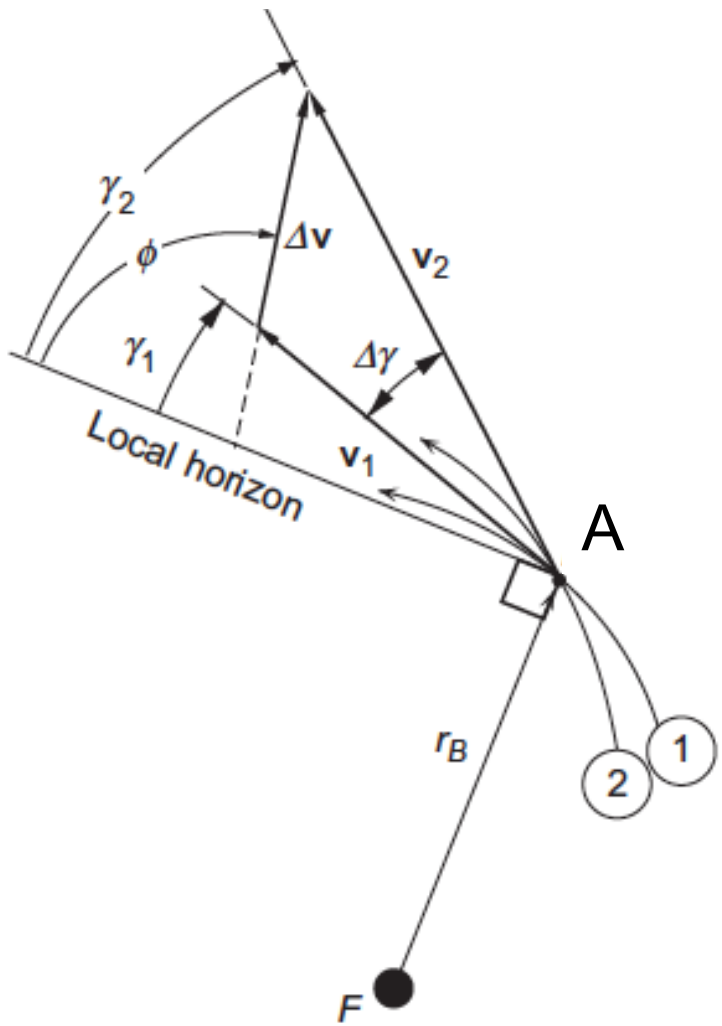
If $r_c/r_a < 11.94$ then Hohmann is more efficient
If $r_c/r_a > 15.58$ then Bi-Elliptic is more efficient



Non-Hohmann Transfers w/ Common Line of Aspides



Non-Hohmann Transfers w/ Common Line of Aspides



$$e_{transfer} = -\frac{r_A - r_B}{r_A \cos\theta_A - r_B \cos\theta_B}$$

$$h_{transfer} = \sqrt{\mu r_A r_B \left(\frac{\cos\theta_A - \cos\theta_B}{r_A \cos\theta_A - r_B \cos\theta_B} \right)}$$

$$\boxed{\Delta V_A = \sqrt{V_1^2 + V_{trans}^2 - 2V_1 V_{trans} \cos\Delta\gamma_A}}$$

$$\tan\phi = \frac{\Delta v_{r@A}}{\Delta v_{\perp@A}} = \frac{v_{r@A}\big|_{orbit2} - v_{r@A}\big|_{orbit1}}{v_{\perp@A}\big|_{orbit2} - v_{\perp@A}\big|_{orbit1}}$$

Non-Hohmann Transfers w/ Common Line of Aspides

$$\Delta V_A = \sqrt{V_1^2 + V_{trans}^2 - 2V_1 V_{trans} \cos \Delta\gamma_A}$$

$$\Delta\gamma_A = \left(\gamma_{trans} - \gamma_1\right)\Big|_{@A}$$

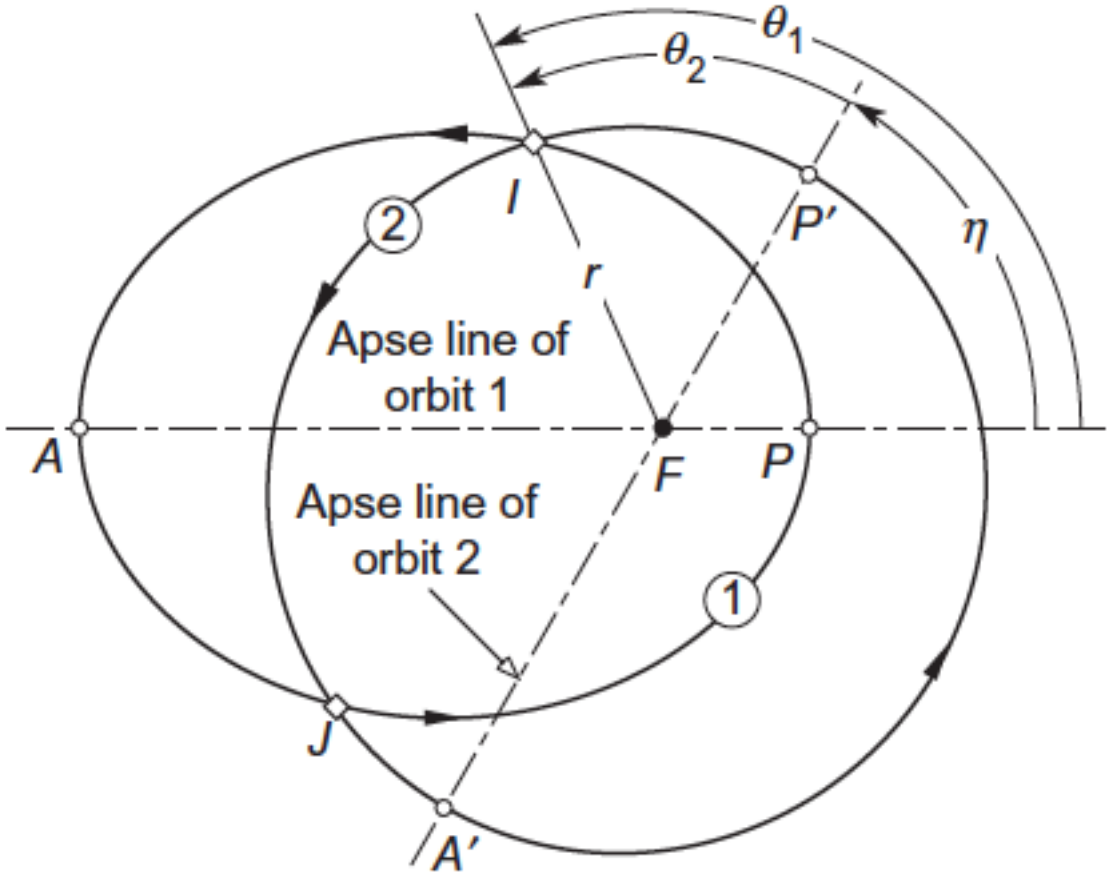
$$\gamma_i = \tan^{-1}\left(\frac{V_r}{V_\perp}\right)_i\Bigg|_{@A}$$

$$\gamma_1 = \tan^{-1}\left(\frac{\left(\mu / h_1\right) e_1 \sin\theta_1}{h_1 / r_A}\right)$$

$$\gamma_{trans} = \tan^{-1}\left(\frac{\left(\mu / h_{trans}\right) e_{trans} \sin\theta_1}{h_{trans} / r_A}\right)$$

Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits



Apse Line Rotation Angle

$$\eta = \theta_1 - \theta_2$$

NOTE:

$$r_{orbit1@I} = r_{orbit2@I}$$

$$\frac{h_1^2 / \mu}{1 + e_1 \cos\theta_1} = \frac{h_2^2 / \mu}{1 + e_2 \cos\theta_2}$$

Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

Setting $\theta_2 = \theta_1 - \eta$ and using the following identity

$$\cos(\theta_1 - \eta) = \cos\theta_1 \cos\eta + \sin\theta_1 \sin\eta$$

Then θ_1 has two solution corresponding to point *I* and *J*

$$\theta_1 = \zeta \pm \cos^{-1}\left(\frac{h_1^2 - h_2^2}{e_1 h_2^2 - e_2 h_1^2 \cos\eta}\cos\zeta\right)$$

$$\zeta = \tan^{-1}\left(\frac{-e_2 h_1^2 \sin\eta}{e_1 h_2^2 - e_2 h_1^2 \cos\eta}\right)$$

Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

Given the orbit information of the two orbits

Next, compute $r,\ v_{\perp i},\ v_{ri},\ \gamma_i$ using θ_1 and $\theta_2 = \theta_1 - \eta$

$$r = \frac{h_1^2 / \mu}{1 + e_1 \cos\theta_1}$$

$$v_{\perp 1} = h_1 / r$$

$$v_{r1} = \left(\mu / h_1\right) e_1 \sin\theta_1$$

$$\gamma_1 = \tan^{-1}\left(v_{r1} / v_{\perp 1}\right)$$

$$v_1 = \sqrt{v_{r1}^2 + v_{\perp 1}^2}$$

$$v_{\perp 2} = h_2 / r$$

$$v_{r2} = \left(\mu / h_2\right) e_2 \sin\theta_2$$

$$\gamma_2 = \tan^{-1}\left(v_{r2} / v_{\perp 2}\right)$$

$$v_2 = \sqrt{v_{r2}^2 + v_{\perp 2}^2}$$

$$\boxed{\Delta v = \sqrt{v_1^2 + v_2^2 - 2 v_1 v_2 \cos\left(\gamma_2 - \gamma_1\right)}}$$

Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

$$\tan\phi = \frac{\Delta v_{r@I}}{\Delta v_{\perp@I}} = \frac{v_{r@I}\big|_{orbit2} - v_{r@I}\big|_{orbit1}}{v_{\perp@I}\big|_{orbit2} - v_{\perp@I}\big|_{orbit1}}$$

If you want to find the effect of a Δv on orbit 1 at θ_1

$$\boxed{\tan\theta_2 = \frac{\left(v_{\perp1} + \Delta v_\perp\right)\left(v_{r1} + \Delta v_r\right)}{\left(v_{\perp1} + \Delta v_\perp\right)^2 e_1 \cos\theta_1 + \left(2v_{\perp1} + \Delta v_\perp\right)\Delta v_\perp} \frac{v_{\perp1}^2}{\left(\mu / r\right)}}$$

If $\theta_1 = v_r = \Delta v_\perp = 0$ then $\tan\eta = -\frac{r v_{\perp1}}{\mu e_1}\Delta v_r$ (at periapsis)

Apse Line Rotation

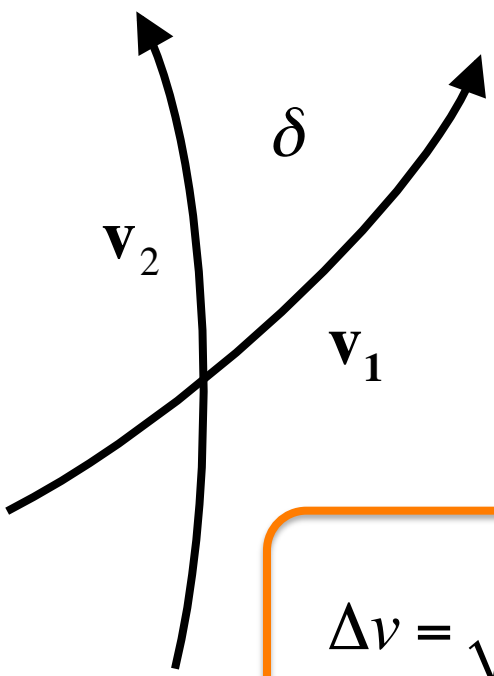
Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

Another useful equation:

$$e_2 = \frac{\left(h_1 + r_1 \Delta v_\perp\right)^2 e_1 \cos\theta_1 + \left(2h_1 + r_1 \Delta v_\perp\right) r_1 \Delta v_\perp}{h_1^2 \cos\theta_2}$$

Plane Change Maneuvers

In general a plane change maneuver changes the orbit plane



$$\Delta\mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = \left(v_{r2} - v_{r1}\right)\hat{u}_r + v_{\perp 2}\hat{u}_{\perp 2} - v_{\perp 1}\hat{u}_{\perp 1}$$

$$\Delta v = \sqrt{\Delta\mathbf{v}\cdot\Delta\mathbf{v}}$$

$$\boxed{\Delta v = \sqrt{\left(v_{r2} - v_{r1}\right)^2 + v_{\perp 1}^2 + v_{\perp 1}^2 - 2v_{\perp 2}v_{\perp 1}\cos\delta}}$$

or

$$\boxed{\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2\left[\cos\Delta\gamma - \cos\gamma_1\cos\gamma_2\left(1 - \cos\delta\right)\right]}}$$

where $\Delta\gamma = \gamma_2 - \gamma_1$

Plane Change Maneuvers

In general a plane change maneuver changes the orbit plane

If there is no plane change, $\delta = 0$ then

$$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos \Delta\gamma}$$

No plane change

and we get the same equation as from slide 130.

If $v_{r1} = v_{r2} = 0$ and $v_{\perp 1} = v_1$ and $v_{\perp 2} = v_2$ then

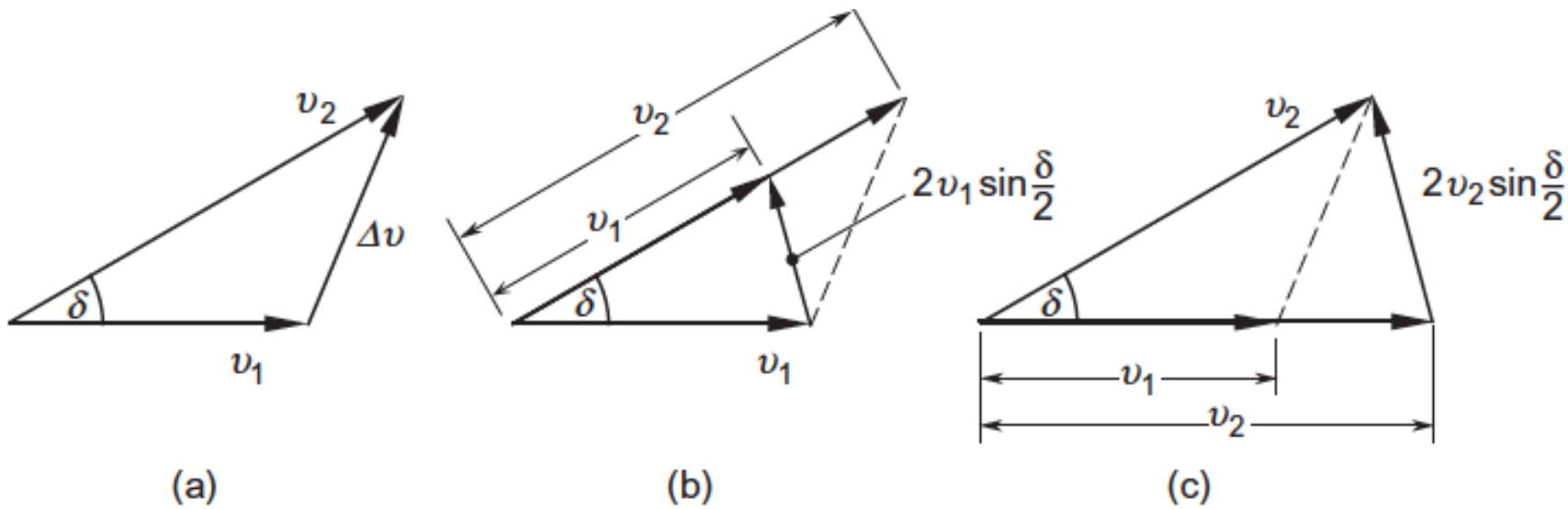
$$\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1 v_2 \cos \delta}$$

Pure plane change (common line of apisdes)

Plane Change

$$\Delta v_{PC} = 2v_C \sin\left(\frac{\delta}{2}\right) = 2v_C \sin\left(\frac{\Delta i}{2}\right)$$

Pure plane change
(circular to circular)



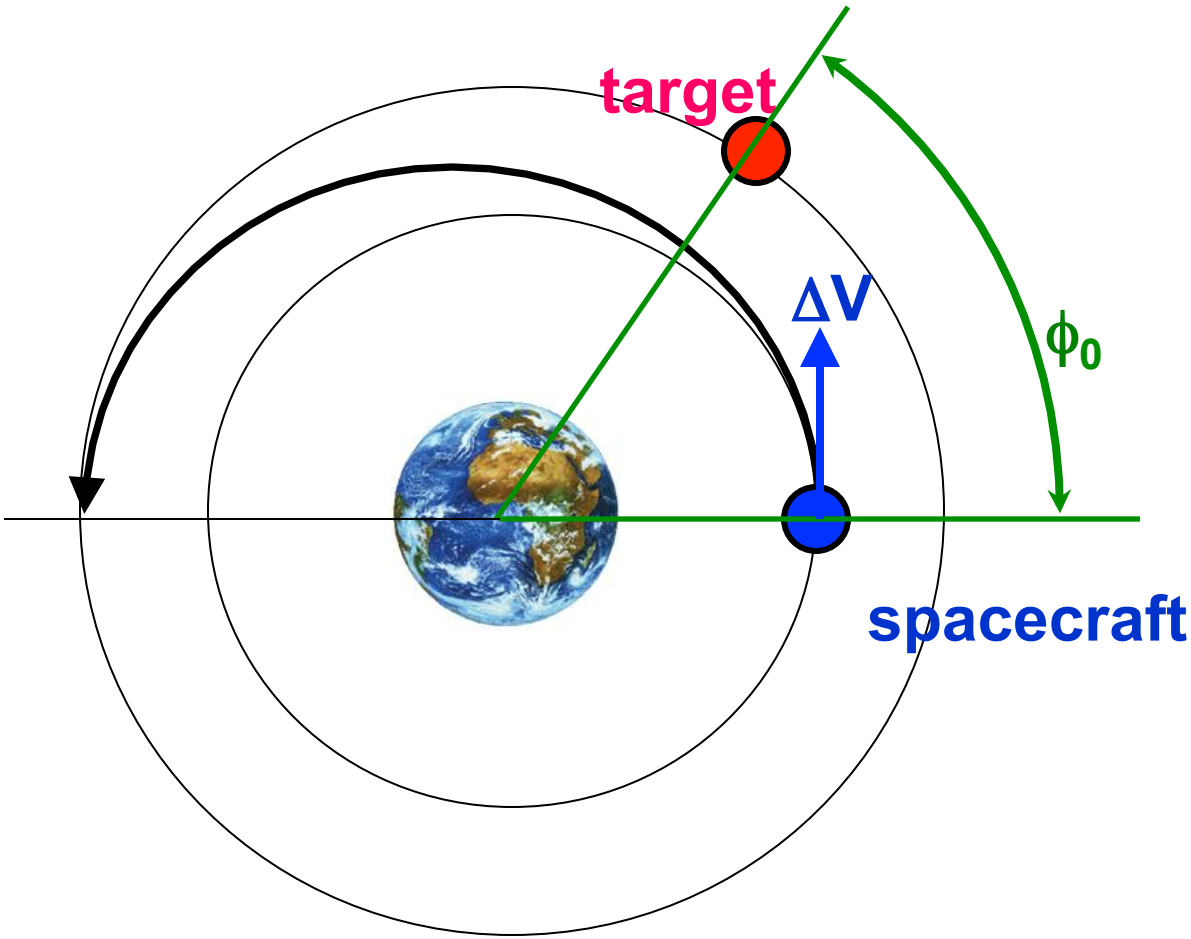
$$\Delta v_{(a)} = \sqrt{\left(v_2 - v_1\right)^2 + 4v_1 v_2 \sin^2\left(\delta/2\right)}$$

$$\Delta v_{(b)} = 2v_1 \sin\left(\delta/2\right) + \left|v_2 - v_1\right|$$

$$\Delta v_{(c)} = \left|v_2 - v_1\right| + 2v_2 \sin\left(\delta/2\right)$$

Rendezvous

Catching a moving target: Hohmann transfer assume the target will be there independent of time, but for rendezvousing the target body is always moving.



TIME = 0

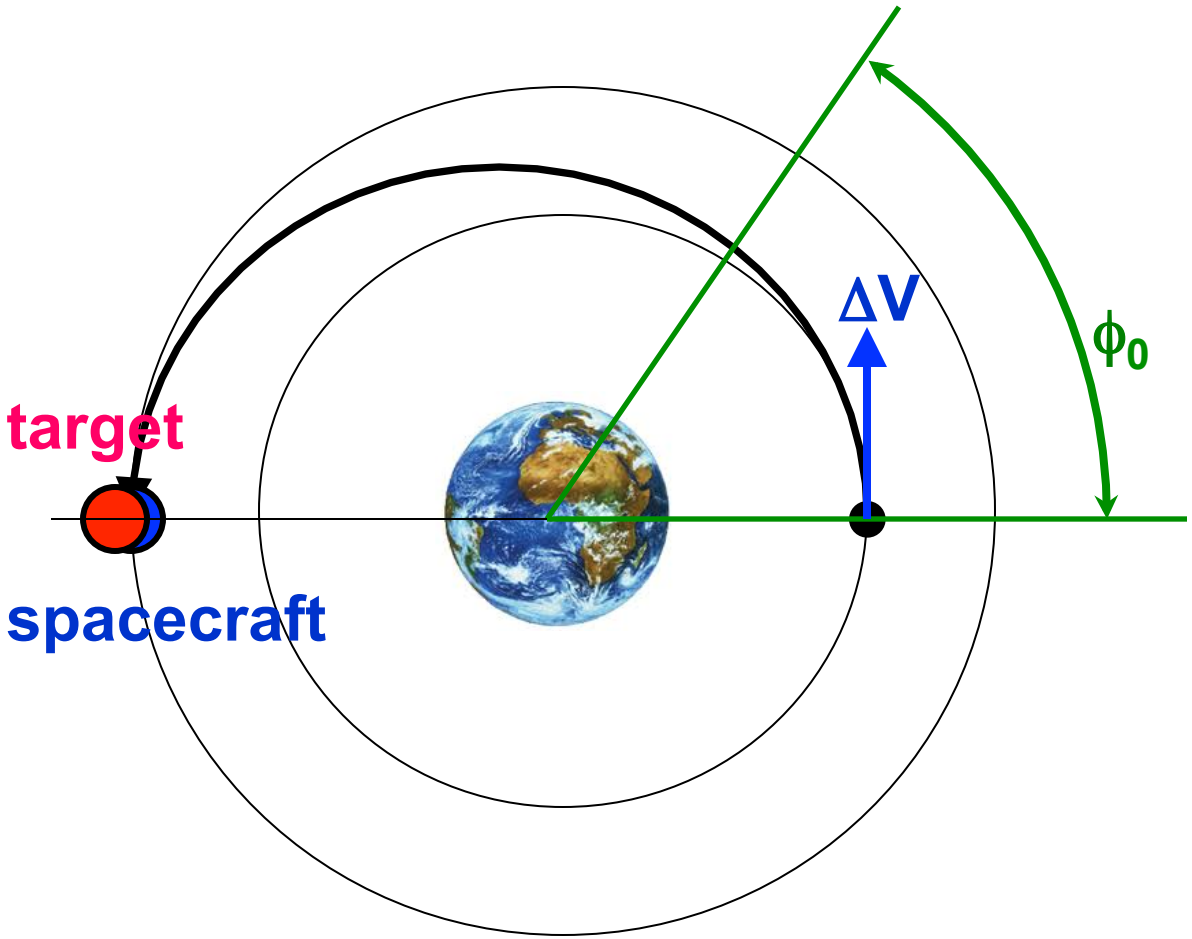
$$\phi_0 = \pi - n_2 \cdot TOF$$

where

$$n_2 = \sqrt{\frac{\mu}{a_2^3}} \equiv \text{mean motion}$$

Rendezvous

Catching a moving target: Hohmann transfer assume the target will be there independent of time, but for rendezvousing the target body is always moving.



TIME = TOF

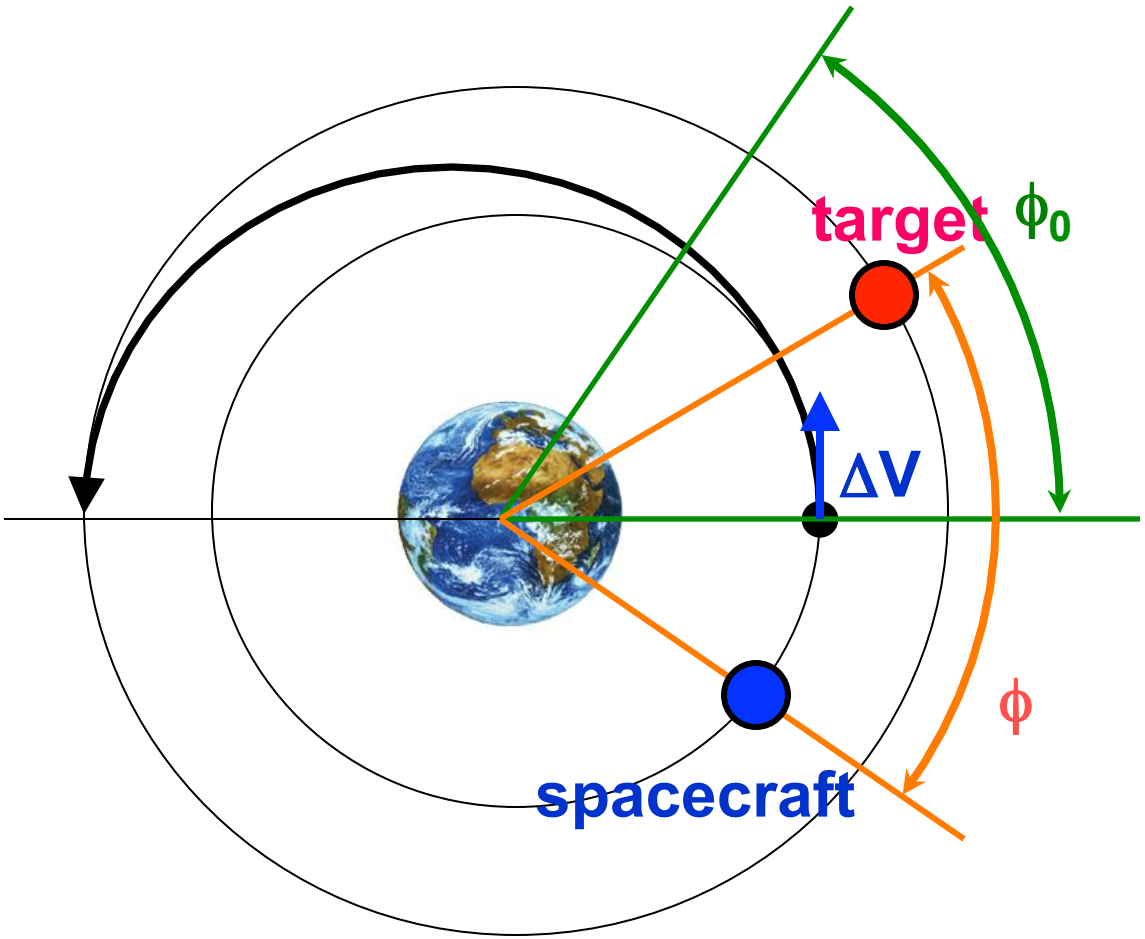
$$\phi_0 = \pi - n_{\text{target}} \cdot TOF$$

where

$$n_{\text{target}} = \sqrt{\frac{\mu}{a_{\text{target}}^3}} \equiv \text{mean motion}$$

Wait Time (WT)

How long do you wait before you can perform (or initiate) the transfer?

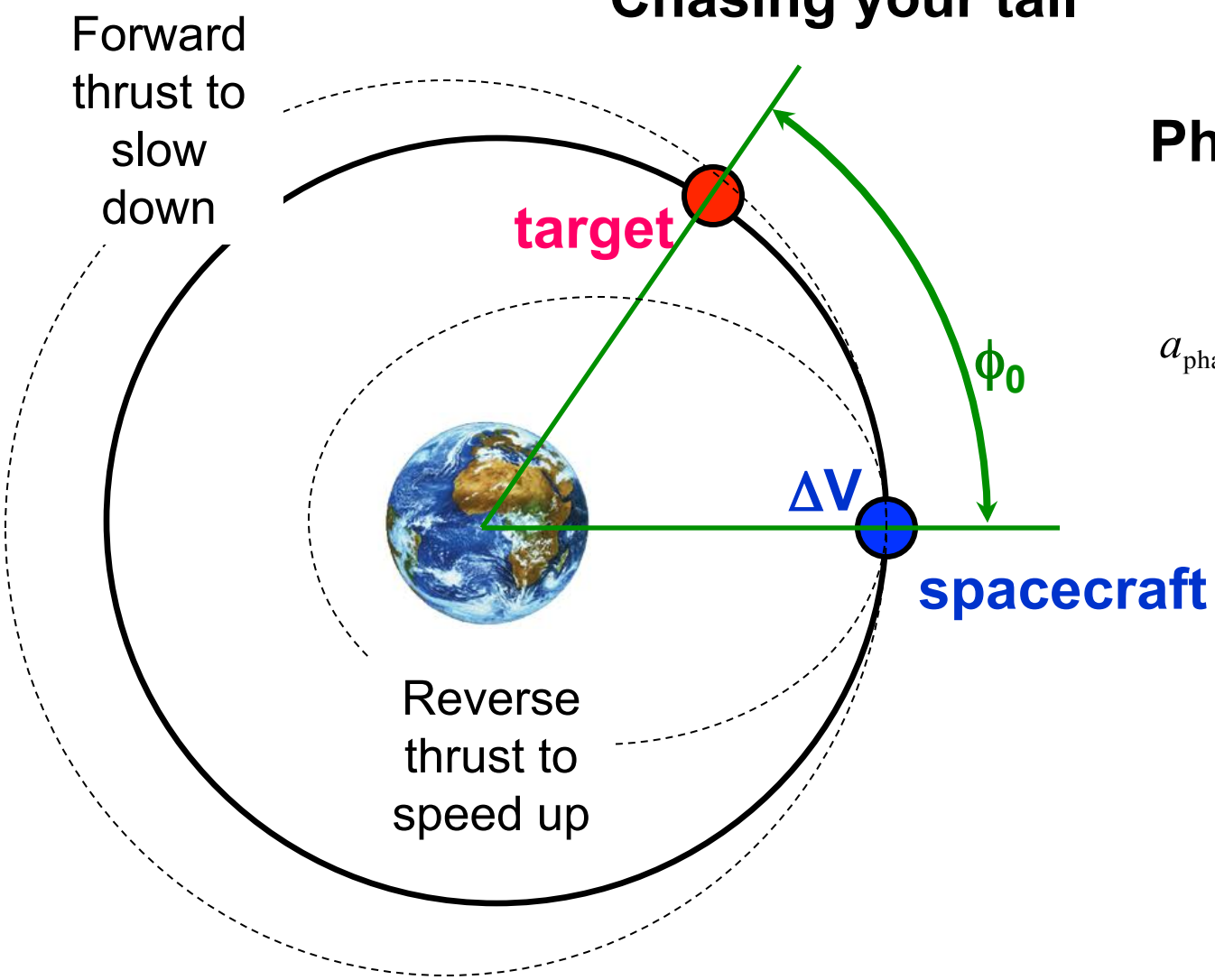


TIME = -WT

$$WT = \frac{\phi - \phi_0}{n_{\text{target}} - n_{sc}}$$

Co-orbital Rendezvous

Chasing your tail

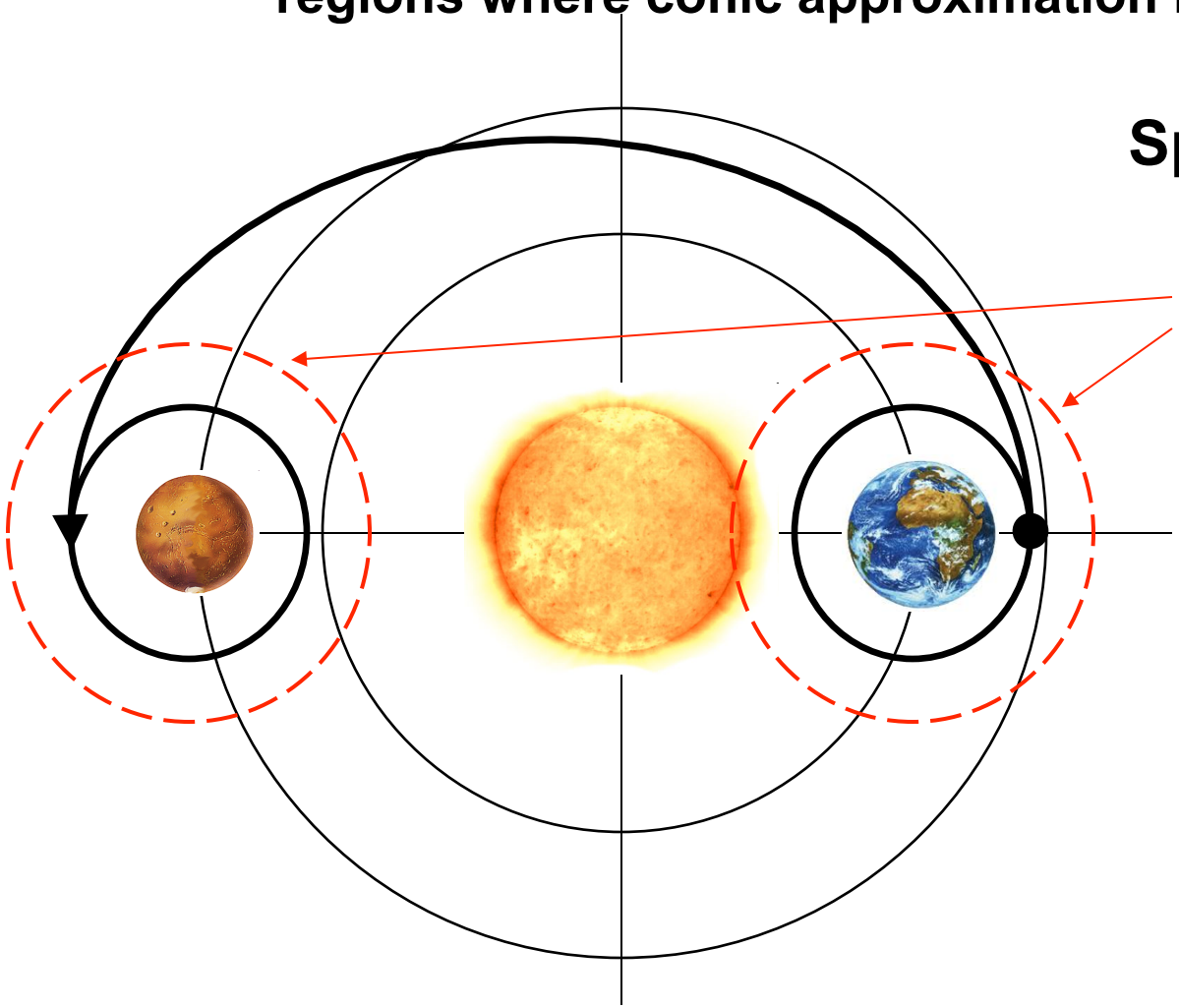


Phasing orbit size

$$a_{\text{phasing}} = \left[\mu \left(\frac{2\pi - \phi_{\text{initial}}}{2\pi \cdot n_{\text{target}}} \right)^2 \right]^{1/3}$$

Patched-Conic

An approximation breaks the <u>interplanetary trajectory</u> into regions where conic approximation is applicable



Sphere of Influence

$$R_{SOI} = a_{planet}\left(\frac{m_{planet}}{m_{Sun}}\right)^{2/5}$$

Patched-Conic

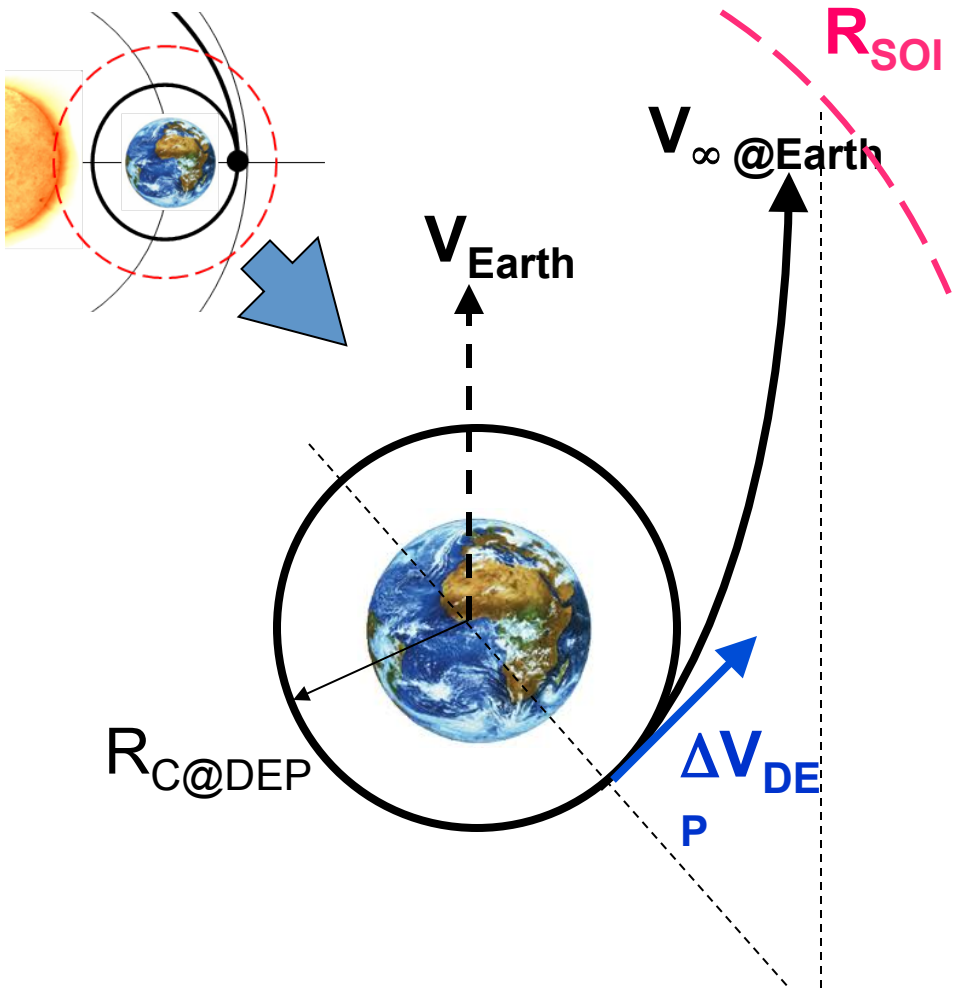
An approximation breaks the <u>interplanetary trajectory</u> into regions where conic approximation is applicable

- Computational Steps
  - PATCH 1: Compute the Hohmann transfer ΔVs
  - PATCH 2: Compute the Launch portion
  - PATCH 3: Compute the Arrival portion

- PATCH 1
  - The ΔV_1 from the interplanetary Hohmann is V_∞ at Earth
  - The ΔV_2 from the interplanetary Hohmann is V_∞ at target body arrival

Patched-Conic

An approximation breaks the <u>interplanetary trajectory</u> into regions where conic approximation is applicable



Earth Departure (assume circular orbit)

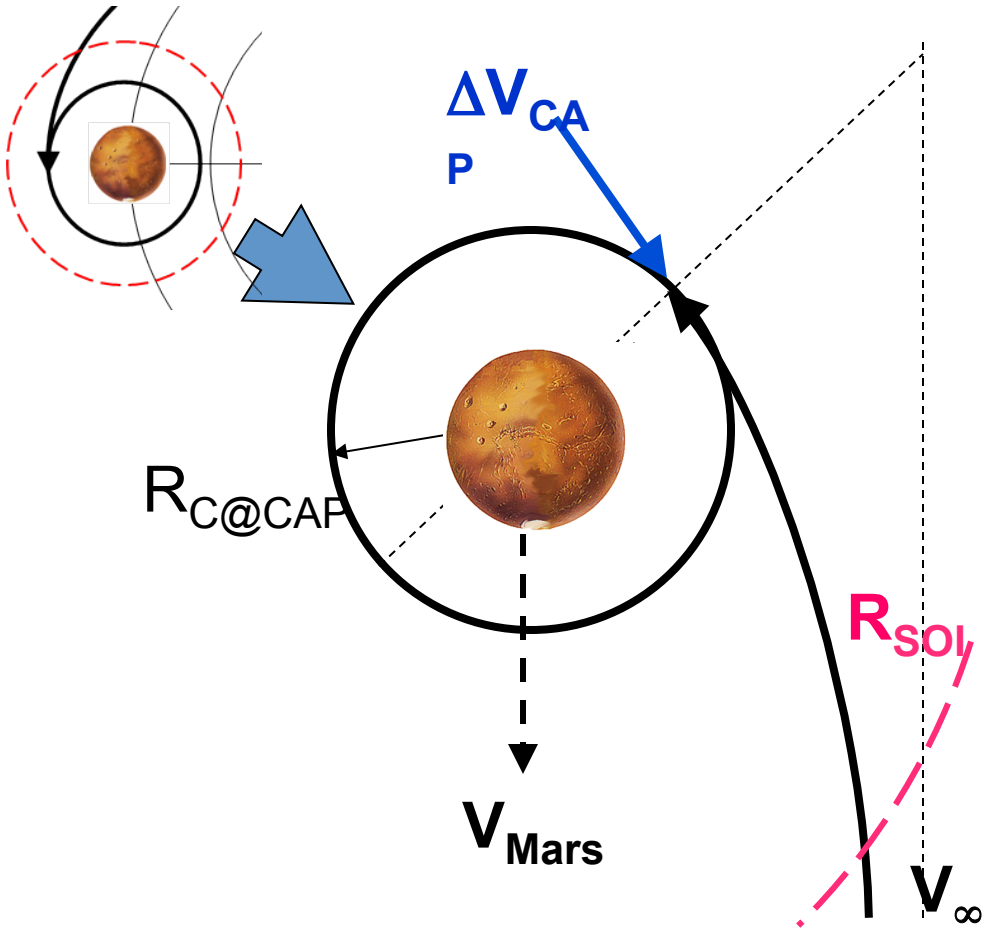
$$\Delta V_{DEP} = V_{C_{DEP}} \left[\sqrt{2 + \left(\frac{V_{\infty @ departure}}{V_{C_{DEP}}} \right)^2} - 1 \right]$$

where

$$V_{C_{DEP}} = \sqrt{\frac{\mu_{DEP}}{R_{C_{DEP}}}}$$

Patched-Conic

An approximation breaks the <u>interplanetary trajectory</u> into regions where conic approximation is applicable



Target Body Capture (assume circular orbit)

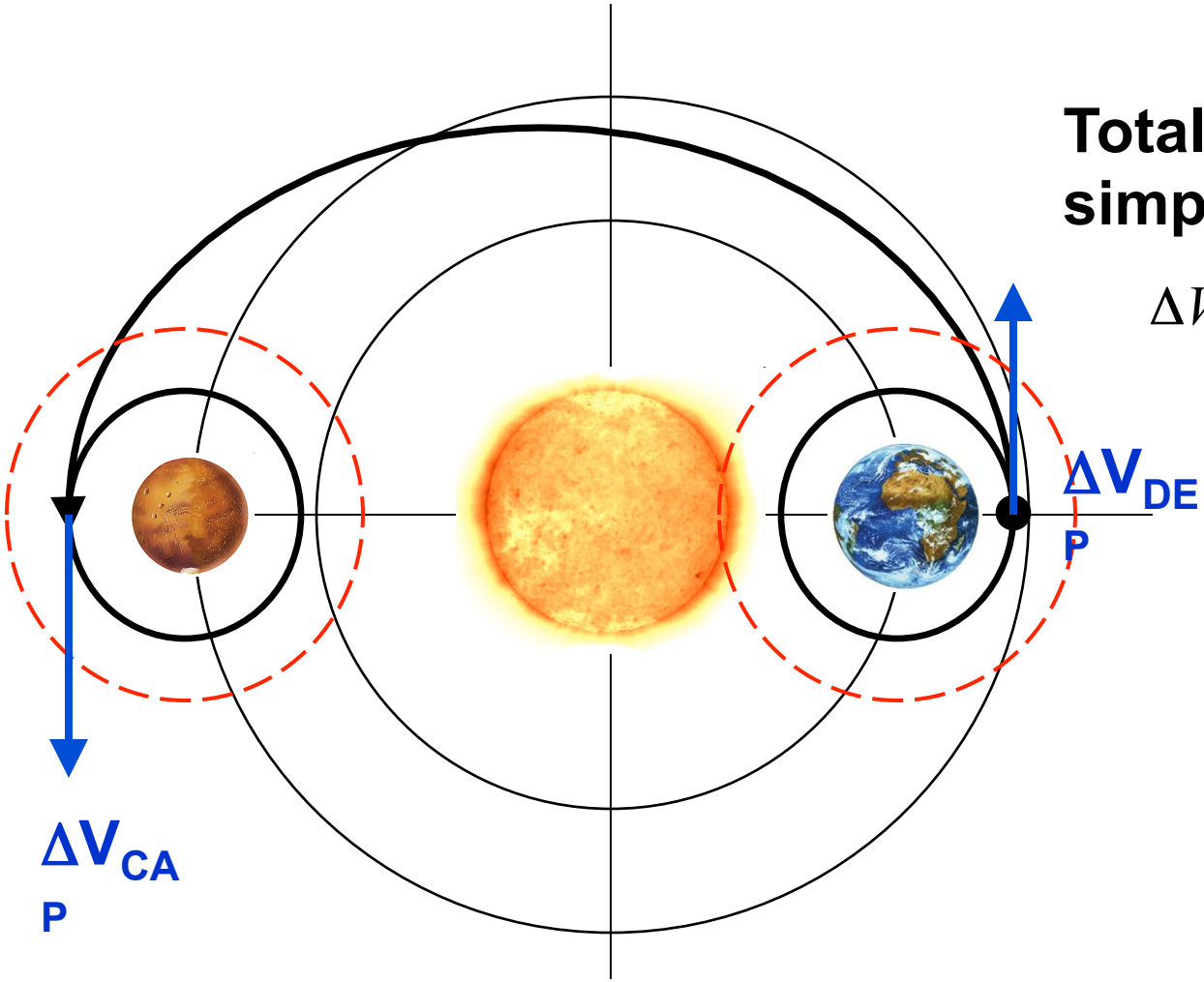
$$\Delta V_{CAP} = V_{C_{CAP}} \left[\sqrt{2 + \left(\frac{V_{\infty @ arrival}}{V_{C_{CAP}}} \right)^2} - 1 \right]$$

where

$$V_{C_{CAP}} = \sqrt{\frac{\mu_{CAP}}{R_{C_{CAP}}}}$$

Patched-Conic

An approximation breaks the <u>interplanetary trajectory</u> into regions where conic approximation is applicable

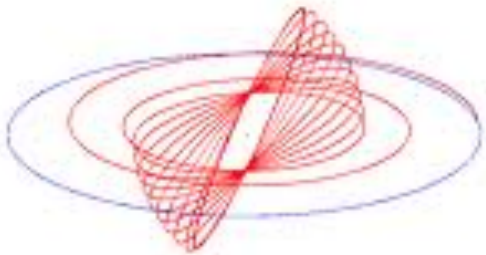


Total Mission ΔV for a simple 2 burn transfer

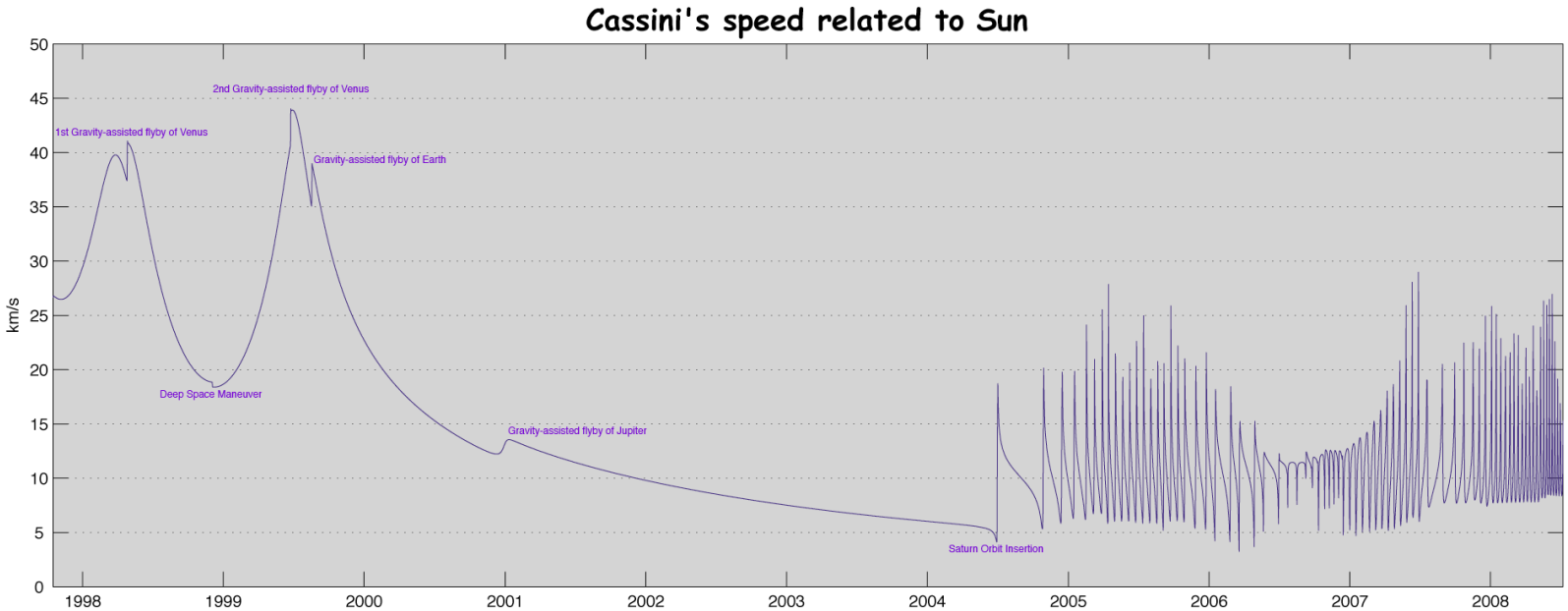
$$\Delta V_{MISSION} = \Delta V_{DEP} + \Delta V_{CAP}$$

Planetary Flyby

Getting a boost

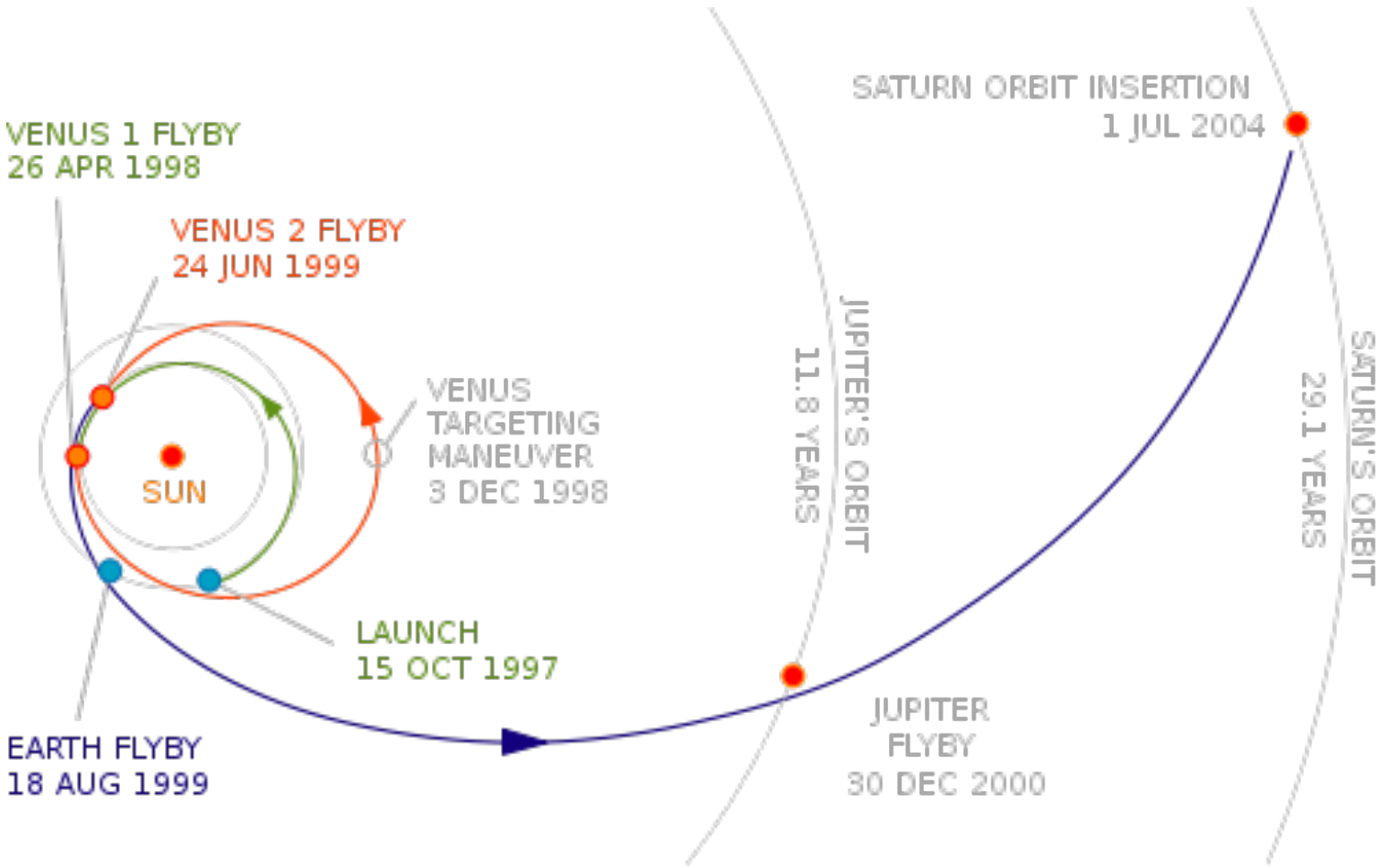


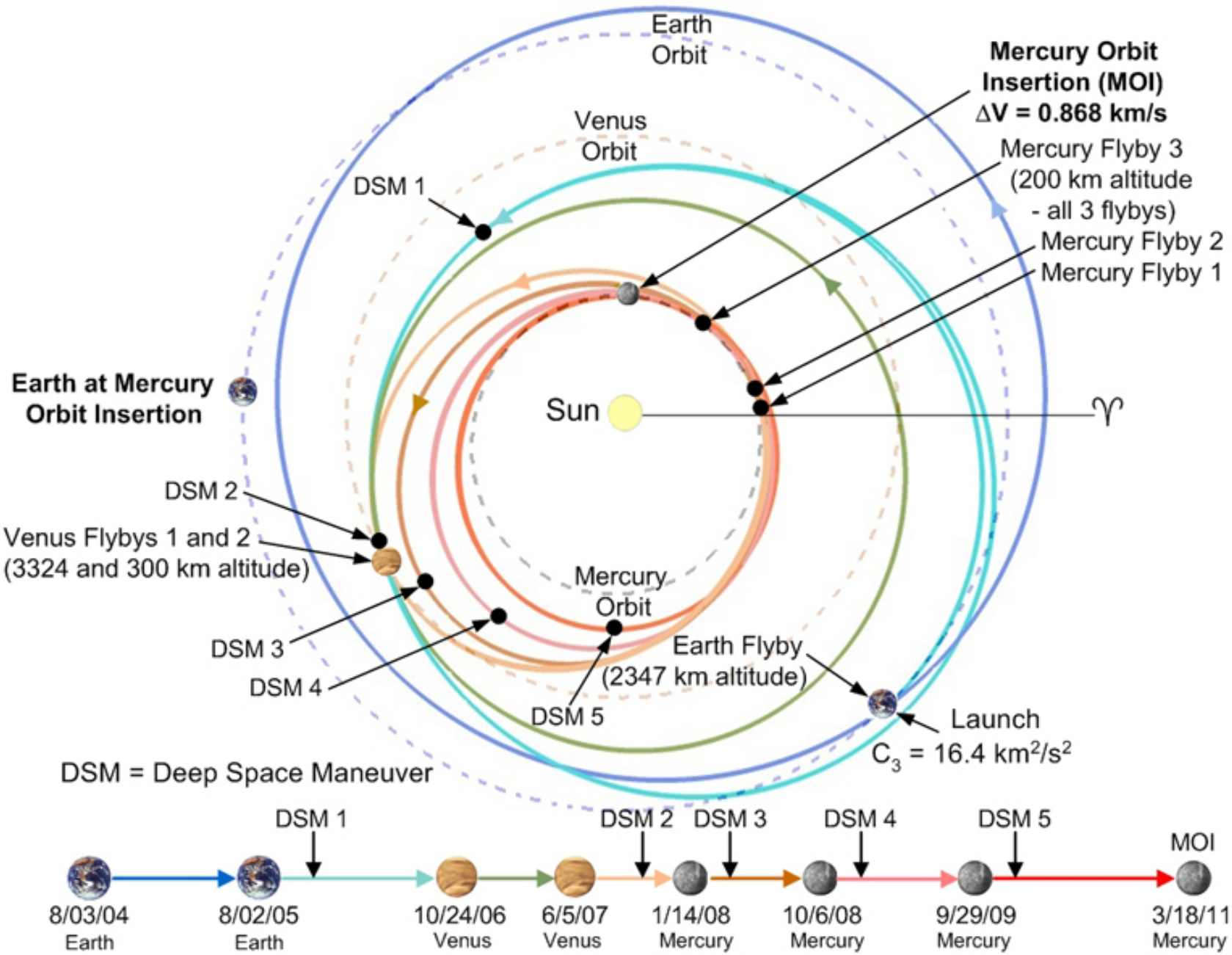
- Gravity-assists or flybys are used to
  - Reduce or increase the heliocentric (wrt. Sun) energy of the spacecraft (orbit pumping), or
  - Change the heliocentric orbit plane of the spacecraft (orbit cranking),



Planetary Flyby

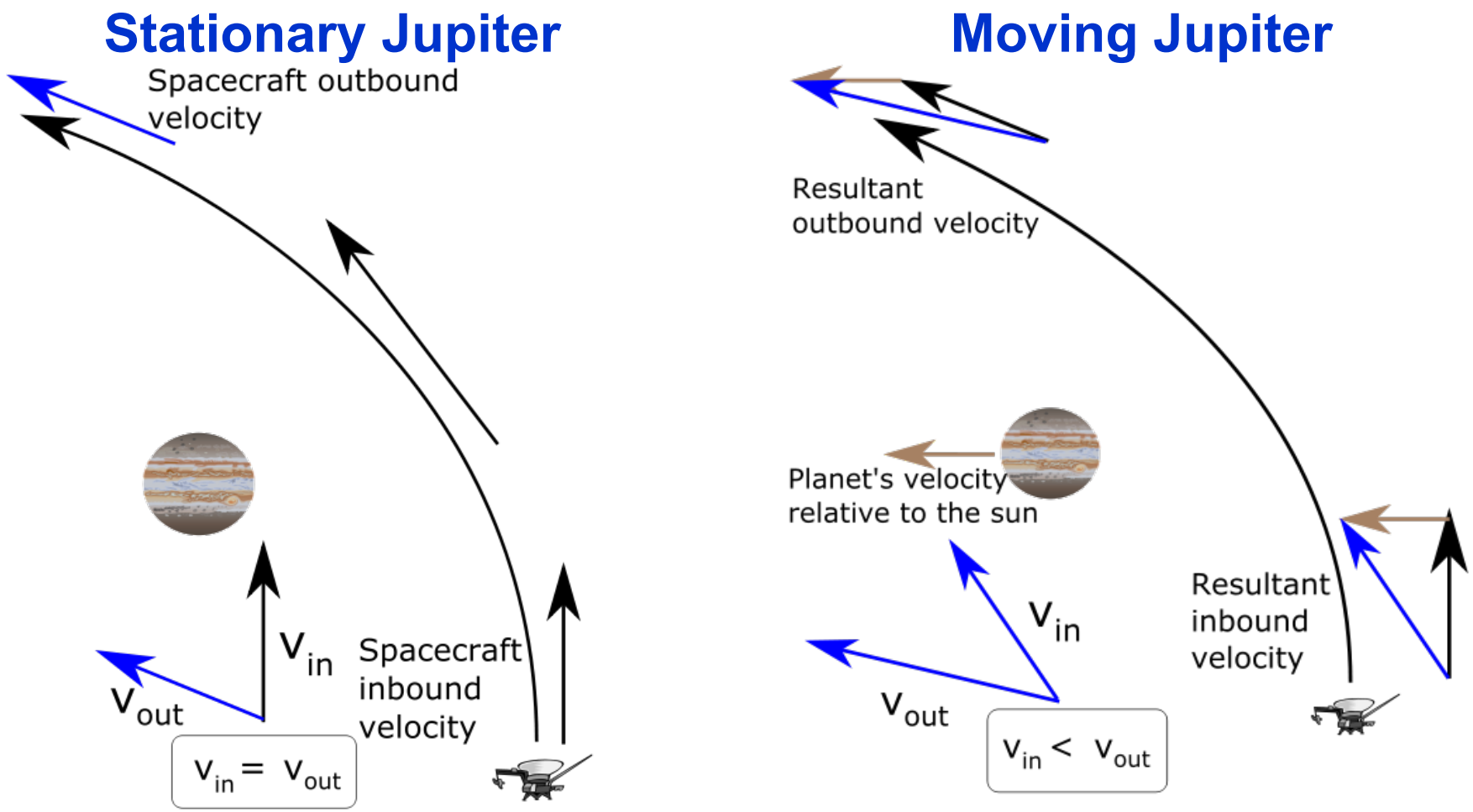
Getting a boost




Earth Orbit
Mercury Orbit Insertion (MOI)
ΔV = 0.868 km/s
Venus Orbit
Mercury Flyby 3 (200 km altitude - all 3 flybys)
DSM 1
Mercury Flyby 2
Mercury Flyby 1
Earth at Mercury Orbit Insertion
Sun
♈
DSM 2
Venus Flybys 1 and 2 (3324 and 300 km altitude)
Mercury Orbit
DSM 3
Earth Flyby (2347 km altitude)
DSM 4
DSM 5
Launch
C_3 = 16.4 km²/s²
DSM = Deep Space Maneuver
DSM 1
DSM 2
DSM 3
DSM 4
DSM 5
MOI
8/03/04 Earth
8/02/05 Earth
10/24/06 Venus
6/5/07 Venus
1/14/08 Mercury
10/6/08 Mercury
9/29/09 Mercury
3/18/11 Mercury

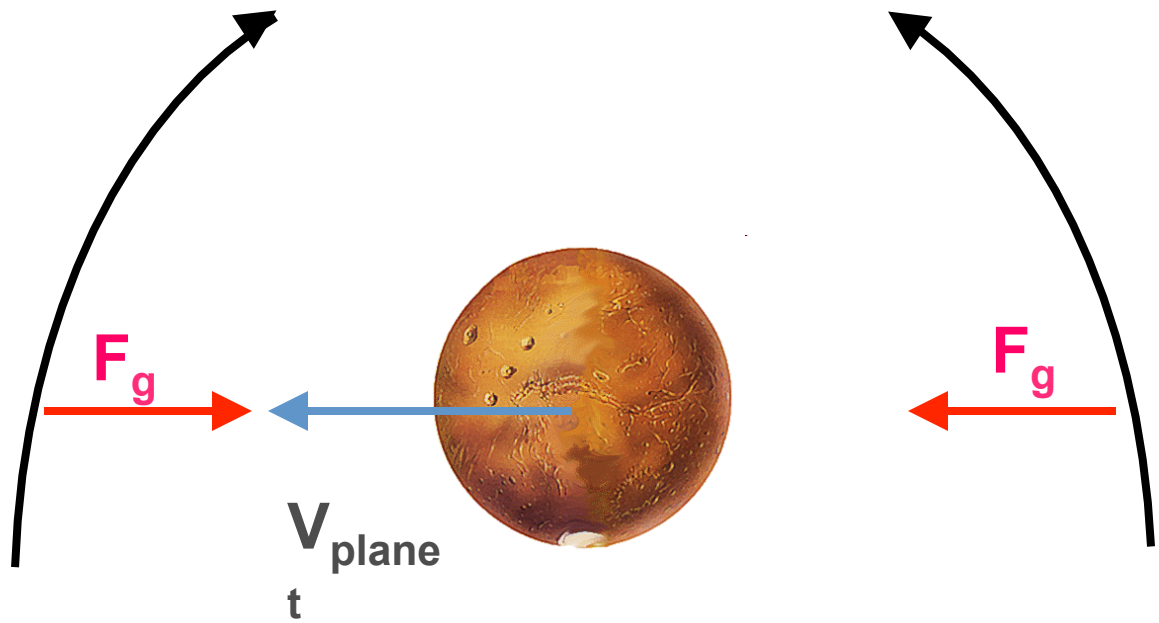
Planetary Flyby

Getting a boost



Planetary Flyby

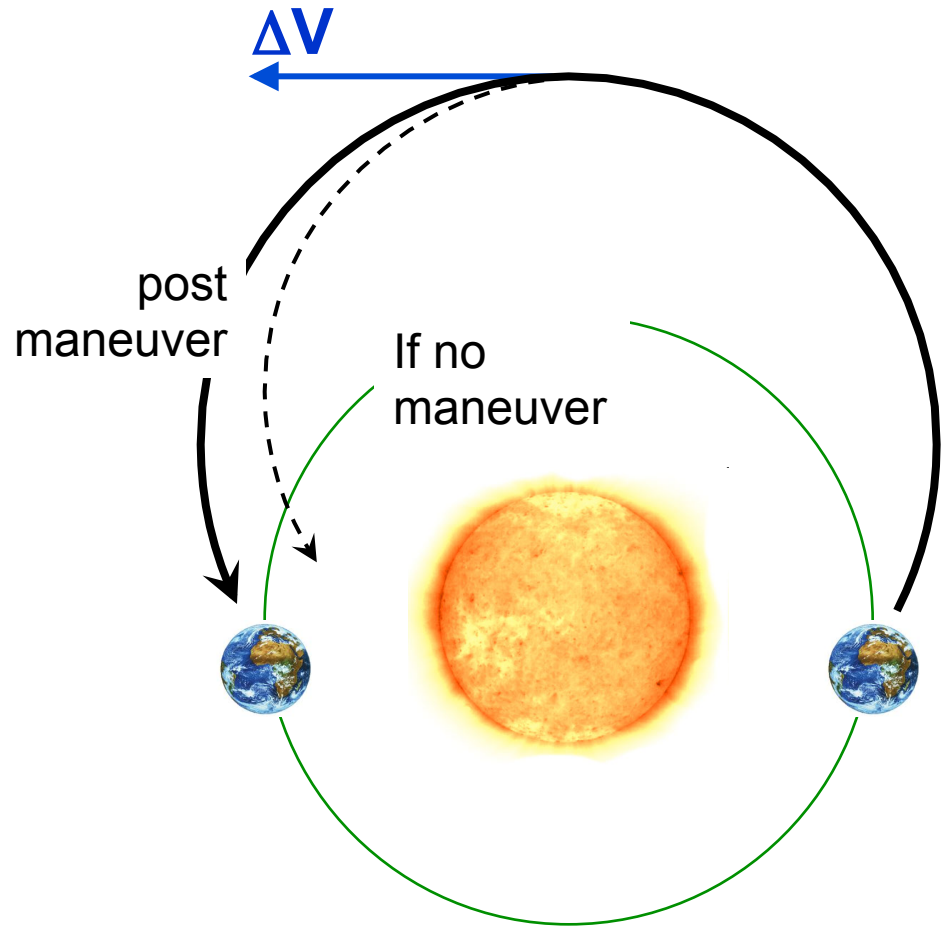
Getting a boost



Decrease energy wrt. Sun

Increases energy wrt. Sun

Planetary Flyby

Getting a boost

- V_∞ leveraging is the use of deep space maneuvers to modify the V_∞ at a body. A typical example is an Earth launch, ΔV maneuver (usually near apoapsis), followed by an Earth gravity assist (ΔV-EGA).