Orbital Maneuvers

Chapter 6
Maneuvers

• We assume our maneuvers are “instantaneous”

• ΔV changes can be applied by changing the magnitude “pump” or by changing the direction “crank”

• Rocket equation: relating ΔV and change in mass

\[
\Delta V = g_0 I_{SP} \ln \left( \frac{m_{initial}}{m_{final}} \right)
\]

or

\[
m_{final} = \frac{m_{initial}}{\exp \left( \frac{\Delta V}{g_0 I_{SP}} \right)}
\]

where \( g_0 = 9.806 \, \text{m/s}^2 \)
$I_{sp}$

- $I_{sp}$ is the specific impulse (units of seconds) and measures the performance of the rocket.

Table 6.1  Some Typical Specific Impulses

<table>
<thead>
<tr>
<th>Propellant</th>
<th>$I_{sp}$ (seconds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cold gas</td>
<td>50</td>
</tr>
<tr>
<td>Monopropellant hydrazine</td>
<td>230</td>
</tr>
<tr>
<td>Solid propellant</td>
<td>290</td>
</tr>
<tr>
<td>Nitric acid/monomethylhydrazine</td>
<td>310</td>
</tr>
<tr>
<td>Liquid oxygen/liquid hydrogen</td>
<td>455</td>
</tr>
<tr>
<td>Ion propulsion</td>
<td>$&gt;3000$</td>
</tr>
</tbody>
</table>
\[ \Delta V = |V_2 - V_1| \]

- \( \Delta V \) ("delta"-V) represents the instantaneous change in velocity (from current velocity to the desired velocity).

Circular Orbit Velocity

\[ V_C = \sqrt{\frac{\mu}{R}} \]

Elliptical Orbit Velocity

\[ V_E = \sqrt{\mu \left( \frac{2}{R} - \frac{1}{a} \right)} \]
Tangent Burns

- Initially you are in a circular orbit with radius $R_1$ around Earth

\[ V_{C_1} = \sqrt{\frac{\mu}{R_1}} \]
Tangent Burns

- Initially you are in a circular orbit with radius $R_1$ around Earth
  $$V_{C_1} = \sqrt{\frac{\mu}{R_1}}$$

- You perform a burn now which puts you into the red-dotted orbit. So you perform a $\Delta V$.
  $$\Delta V = |V_{E_1} - V_{C_1}|$$

Your new orbit after the $\Delta V$ burn
Tangent Burns

- Initially you are in a circular orbit with radius $R_1$ around Earth

\[ V_{C_1} = \sqrt{\frac{\mu}{R_1}} \]

- You perform a burn now which puts in into the red-dotted orbit. So you perform a $\Delta V$.

\[ \Delta V = |V_{E_1} - V_{C_1}| \]

where

\[ V_{E_1} = \sqrt{\mu \left( \frac{2}{R_1} - \frac{1}{a} \right)} \quad \text{and} \quad a = \frac{1}{2} \left( R_1 + R_2 \right) \]
The most efficient 2-burnmaneuver to transfer between 2 co-planar circular orbits.

\[ \Delta V = \Delta V_1 + \Delta V_2 \]
\[ \Delta V_1 = \sqrt{\frac{\mu}{R_1}} \left( \sqrt{\frac{2R_2}{R_1 + R_2}} - 1 \right) \]
\[ \Delta V_2 = \sqrt{\frac{\mu}{R_2}} \left( 1 - \sqrt{\frac{2R_1}{R_1 + R_2}} \right) \]

\[ TOF = \frac{1}{2} P = \pi \sqrt{\frac{a^3}{\mu}} \]
Hohmann Transfer (non-circular)

- Another way to view it is using angular momentum

\[
h = \sqrt{2\mu \left( \frac{r_a r_p}{r_a + r_p} \right)}
\]

\[
\Delta V_{\text{total}_{-3}} = \Delta V_A + \Delta V_B
\]

\[
\Delta V_{\text{total}_{-3'}} = \Delta V_{A'} + \Delta V_{B'}
\]
Hohmann Transfer (non-circular)

\[ h_1 = \sqrt{2\mu \left( \frac{r_A r_{A'}}{r_A + r_{A'}} \right)} \]

\[ V_{A|1} = \frac{h_1}{r_A} \]

\[ V_{A|3} = \frac{h_3}{r_A} \]

\[ h_2 = \sqrt{2\mu \left( \frac{r_B r_{B'}}{r_B + r_{B'}} \right)} \]

\[ V_{B|1} = \frac{h_2}{r_B} \]

\[ V_{B|3} = \frac{h_3}{r_B} \]

\[ h_3 = \sqrt{2\mu \left( \frac{r_A r_B}{r_A + r_B} \right)} \]

\[ \Delta V_A = abs \left( V_{A|3} - V_{A|1} \right) \]

\[ \Delta V_{A'} = abs \left( V_{A'|3} - V_{A'|1} \right) \]

\[ h_3' = \sqrt{2\mu \left( \frac{r_{A'} r_{B'}}{r_{A'} + r_{B'}} \right)} \]

\[ V_{B'|1} = \frac{h_2}{r_{B'}} \]

\[ V_{B'|3} = \frac{h_3}{r_{B'}} \]

\[ \Delta V_B = abs \left( V_{B|2} - V_{B|3} \right) \]

\[ \Delta V_{B'} = abs \left( V_{B'|2} - V_{B'|3} \right) \]
Bi-Elliptic Transfer
Bi-Elliptic Transfer

If \( r_c/r_a < 11.94 \) then Hohmann is more efficient
If \( r_c/r_a > 15.58 \) then Bi-Elliptic is more efficient
Non-Hohmann Transfers w/ Common Line of Aspides
Non-Hohmann Transfers w/ Common Line of Aspides

\[ e_{\text{transfer}} = - \frac{r_A - r_B}{r_A \cos \theta_A - r_B \cos \theta_B} \]

\[ h_{\text{transfer}} = \sqrt{\mu r_A r_B \left( \frac{\cos \theta_A - \cos \theta_B}{r_A \cos \theta_A - r_B \cos \theta_B} \right)} \]

\[ \Delta V_A = \sqrt{V_1^2 + V_{\text{trans}}^2 - 2V_1 V_{\text{trans}} \cos \Delta \gamma_A} \]

\[ \tan \phi = \frac{\Delta v_{r@A}}{\Delta v_{\perp A}} = \frac{v_{r@A}|_{\text{orbit}2} - v_{r@A}|_{\text{orbit}1}}{v_{\perp A}|_{\text{orbit}2} - v_{\perp A}|_{\text{orbit}1}} \]
Non-Hohmann Transfers w/ Common Line of Aspides

$$\Delta V_A = \sqrt{V_1^2 + V_{trans}^2} - 2V_1V_{trans} \cos \Delta \gamma_A$$

$$\Delta \gamma_A = (\gamma_{trans} - \gamma_1)|_{@A}$$

$$\gamma_i = \tan^{-1} \left( \frac{V_r}{V_{\perp}} \right)|_{@A}$$

$$\gamma_1 = \tan^{-1} \left( \frac{(\mu / h_1)e_1 \sin \theta_1}{h_1 / r_A} \right)$$

$$\gamma_{trans} = \tan^{-1} \left( \frac{(\mu / h_{trans})e_{trans} \sin \theta_1}{h_{trans} / r_A} \right)$$
Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

Apse Line Rotation Angle

$$\eta = \theta_1 - \theta_2$$

NOTE:

$$\frac{r_{\text{orbit1}@I}}{r_{\text{orbit2}@I}} = \frac{h_1^2 / \mu}{1 + e_1 \cos \theta_1} = \frac{h_2^2 / \mu}{1 + e_2 \cos \theta_2}$$
Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

Setting \( \theta_2 = \theta_1 - \eta \) and using the following identity

\[
\cos(\theta_1 - \eta) = \cos \theta_1 \cos \eta + \sin \theta_1 \sin \eta
\]

Then \( \theta_1 \) has two solutions corresponding to point I and J

\[
\theta_1 = \xi \pm \cos^{-1} \left( \frac{h_1^2 - h_2^2}{e_1 h_2^2 - e_2 h_1^2 \cos \eta} \cos \xi \right)
\]

\[
\xi = \tan^{-1} \left( \frac{-e_2 h_1^2 \sin \eta}{e_1 h_2^2 - e_2 h_1^2 \cos \eta} \right)
\]
Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits.

Given the orbit information of the two orbits
Next, compute \( r, v_{\perp i}, v_{ri}, \gamma_i \) using \( \theta_1 \) and \( \theta_2 = \theta_1 - \eta \)

\[
\begin{align*}
    r &= \frac{h_1^2 / \mu}{1 + e_1 \cos \theta_1} \\
    v_{\perp 1} &= \frac{h_1}{r} \\
    v_{r1} &= \left( \frac{\mu}{h_1} \right) e_1 \sin \theta_1 \\
    \gamma_1 &= \tan^{-1} \left( \frac{v_{r1}}{v_{\perp 1}} \right) \\
    v_1 &= \sqrt{v_{r1}^2 + v_{\perp 1}^2} \\
    v_{\perp 2} &= \frac{h_2}{r} \\
    v_{r2} &= \left( \frac{\mu}{h_2} \right) e_2 \sin \theta_2 \\
    \gamma_2 &= \tan^{-1} \left( \frac{v_{r2}}{v_{\perp 2}} \right) \\
    v_2 &= \sqrt{v_{r2}^2 + v_{\perp 2}^2} \\
    \Delta v &= \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos(\gamma_2 - \gamma_1)}
\end{align*}
\]
Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

$$\tan \phi = \frac{\Delta v_{r@I}}{\Delta v_{\perp@I}} = \frac{v_{r@I}|_{\text{orbit\,2}} - v_{r@I}|_{\text{orbit\,1}}}{v_{\perp@I}|_{\text{orbit\,2}} - v_{\perp@I}|_{\text{orbit\,1}}}$$

If you want to find the effect of a \( \Delta v \) on orbit 1 at \( \theta_1 \)

$$\tan \theta_2 = \frac{(v_{\perp1} + \Delta v_{\perp})(v_{r1} + \Delta v_r)}{(v_{\perp1} + \Delta v_{\perp})^2 e_1 \cos \theta_1 + (2v_{\perp1} + \Delta v_{\perp})\Delta v_{\perp}} \frac{v_{\perp1}^2}{\mu / r}$$

If \( \theta_1 = v_r = \Delta v_{\perp} = 0 \) then \( \tan \eta = -\frac{rv_{\perp1}}{\mu e_1} \Delta v_r \) (at periapsis)
Apse Line Rotation

Opportunity to transfer from one orbit to another using a single maneuver occurs at intersection points of the orbits

Another useful equation:

\[
e_2 = \frac{(h_1 + r_1 \Delta v_\perp)^2 e_1 \cos \theta_1 + (2h_1 + r_1 \Delta v_\perp) r_1 \Delta v_\perp}{h_1^2 \cos \theta_2}
\]
Plane Change Maneuvers

In general a plane change maneuver changes the orbit plane

\[
\Delta v = \mathbf{v}_2 - \mathbf{v}_1 = (v_{r2} - v_{r1}) \hat{u}_r + v_{\perp2} \hat{u}_{\perp2} - v_{\perp1} \hat{u}_{\perp1}
\]

\[
\Delta v = \sqrt{\Delta \mathbf{v} \cdot \Delta \mathbf{v}}
\]

\[
\Delta v = \sqrt{(v_{r2} - v_{r1})^2 + v_{\perp1}^2 + v_{\perp1}^2 - 2v_{\perp2}v_{\perp1} \cos \delta}
\]

or

\[
\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \left[ \cos \Delta \gamma - \cos \gamma_1 \cos \gamma_2 (1 - \cos \delta) \right]}
\]

where \( \Delta \gamma = \gamma_2 - \gamma_1 \)
Plane Change Maneuvers

In general a plane change maneuver changes the orbit plane.

If there is no plane change, \( \delta = 0 \) then

\[
\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos \Delta \gamma}
\]

and we get the same equation as from slide 130.

If \( v_{r1} = v_{r2} = 0 \) and \( v_{\perp1} = v_1 \) and \( v_{\perp2} = v_2 \) then

\[
\Delta v = \sqrt{v_1^2 + v_2^2 - 2v_1v_2 \cos \delta}
\]

No plane change

Pure plane change (common line of apsides)
Plane Change

Pure plane change (circular to circular)

\[ \Delta v_{PC} = 2v_C \sin \left( \frac{\delta}{2} \right) = 2v_C \sin \left( \frac{\Delta i}{2} \right) \]

\[ \Delta v_{(a)} = \sqrt{\left( v_2 - v_1 \right)^2 + 4v_1v_2 \sin^2 \left( \frac{\delta}{2} \right)} \]

\[ \Delta v_{(b)} = 2v_1 \sin \left( \frac{\delta}{2} \right) + |v_2 - v_1| \]

\[ \Delta v_{(c)} = |v_2 - v_1| + 2v_2 \sin \left( \frac{\delta}{2} \right) \]
Rendezvous

Catching a moving target: Hohmann transfer assume the target will be there independent of time, but for rendezvousing the target body is always moving.

\[ \phi_0 = \pi - n_2 \cdot TOF \]

where

\[ n_2 = \sqrt{\frac{\mu}{a_2^3}} \equiv \text{mean motion} \]
Rendezvous

Catching a moving target: Hohmann transfer assume the target will be there independent of time, but for rendezvousing the target body is always moving.

\[
\phi_0 = \pi - n_{\text{target}} \cdot \text{TOF}
\]

where

\[
n_{\text{target}} = \sqrt{\frac{\mu}{a_{\text{target}}^3}} = \text{mean motion}
\]
Wait Time (WT)
How long do you wait before you can perform (or initiate) the transfer?

\[ \text{TIME} = -WT \]

\[ WT = \frac{\phi - \phi_0}{n_{\text{target}} - n_{\text{sc}}} \]
Co-orbital Rendezvous
Chasing your tail

Forward thrust to slow down

Reverse thrust to speed up

target

$\Delta V$

spacecraft

$\phi_0$

Phasing orbit size

$$a_{\text{phasing}} = \left[ \mu \left( \frac{2\pi - \phi_{\text{initial}}}{2\pi \cdot n_{\text{target}}} \right)^2 \right]^{1/3}$$
Patched-Conic

An approximation breaks the interplanetary trajectory into regions where conic approximation is applicable

Sphere of Influence

\[ R_{SOI} = a_{\text{planet}} \left( \frac{m_{\text{planet}}}{m_{\text{Sun}}} \right)^{2/5} \]
Patched-Conic

An approximation breaks the interplanetary trajectory into regions where conic approximation is applicable

• Computational Steps
  – PATCH 1: Compute the Hohmann transfer $\Delta V_s$
  – PATCH 2: Compute the Launch portion
  – PATCH 3: Compute the Arrival portion

• PATCH 1
  – The $\Delta V_1$ from the interplanetary Hohmann is $V_\infty$ at Earth
  – The $\Delta V_2$ from the interplanetary Hohmann is $V_\infty$ at target body arrival
Patched-Conic

An approximation breaks the interplanetary trajectory into regions where conic approximation is applicable.

Earth Departure (assume circular orbit)

\[
\Delta V_{\text{DEP}} = V_{C_{\text{DEP}}} \left[ \sqrt{2 + \left( \frac{V_{\infty @\text{departure}}}{V_{C_{\text{DEP}}}} \right)^2} - 1 \right]
\]

where

\[
V_{C_{\text{DEP}}} = \sqrt{\frac{\mu_{\text{DEP}}}{R_{C_{\text{DEP}}}}}
\]
Patched-Conic

An approximation breaks the interplanetary trajectory into regions where conic approximation is applicable.

Target Body Capture (assume circular orbit)

\[
\Delta V_{CAP} = V_{CAP} \left[ \sqrt{2 + \left( \frac{V_{\infty @arrival}}{V_{CAP}} \right)^2} - 1 \right]
\]

where

\[
V_{CAP} = \sqrt{\frac{\mu_{CAP}}{R_{CAP}}}
\]

\[
\Delta V_{CA} = V_{CA}\]

\[
R_{C@CAP}
\]

\[
V_{Mars}
\]

\[
V_{\infty @Mars}
\]
Patched-Conic

An approximation breaks the interplanetary trajectory into regions where conic approximation is applicable

Total Mission $\Delta V$ for a simple 2 burn transfer

$$\Delta V_{MISSION} = \Delta V_{DEP} + \Delta V_{CAP}$$
Planetary Flyby
Getting a boost

- Gravity-assists or flybys are used to
  - Reduce or increase the heliocentric (wrt. Sun) energy of the spacecraft (orbit pumping), or
  - Change the heliocentric orbit plane of the spacecraft (orbit cranking),

Cassini's speed related to Sun
Planetary Flyby
Getting a boost

VENUS 1 FLYBY
26 APR 1998

VENUS 2 FLYBY
24 JUN 1999

VENUS TARGETING MANEUVER
3 DEC 1998

SUN

VENUS

LAUNCH
15 OCT 1997

EARTH FLYBY
18 AUG 1999

JUPITER'S ORBIT
11.8 YEARS

JUPITER FLYBY
30 DEC 2000

SATURN'S ORBIT
29.1 YEARS

SATURN ORBIT INSERTION
1 JUL 2004
Planetary Flyby
Getting a boost

Stationary Jupiter
- Spacecraft outbound velocity
- \( V_{\text{in}} = V_{\text{out}} \)

Moving Jupiter
- Resultant outbound velocity
- Planet's velocity relative to the sun
- \( V_{\text{in}} < V_{\text{out}} \)
Planetary Flyby

Getting a boost

Decrease energy wrt. Sun

Increases energy wrt. Sun
Planetary Flyby

Getting a boost

- $V_\infty$ leveraging is the use of deep space maneuvers to modify the $V_\infty$ at a body. A typical example is an Earth launch, $\Delta V$ maneuver (usually near apoapsis), followed by an Earth gravity assist ($\Delta V$-EGA).